Solving Radical Equations
ApexMath

Notes

The Big Idea

A **radical equation** is any equation that has a variable inside a radical symbol. For example:

$$\sqrt{x} = 5 \qquad \sqrt{x+3} = 4 \qquad \sqrt{2x-1} = 3$$

To solve a radical equation, the goal is to **isolate the radical** on one side of the equation, and then **eliminate the radical** by raising both sides to the appropriate power.

For a square root, you square both sides. For a cube root, you cube both sides. The key rule is:

$$\text{If } \sqrt{x} = a, \text{ then } \left(\sqrt{x}\right)^2 = a^2, \text{ so } x = a^2$$

The Method — Step by Step

- Step 1: Isolate the radical on one side of the equation. If there are other terms, move them to the other side first.
- Step 2: Square both sides to eliminate the square root. If the equation has a cube root, cube both sides instead.
- Step 3: Solve the resulting equation.
- Step 4: **Check your answer** by substituting it back into the original equation.

Step 4 is not optional. It is a required part of every solution.

Why Must You Always Check?

When you square both sides of an equation, you can accidentally introduce a solution that satisfies the squared equation but not the original. These fake solutions are called **extraneous roots**.

For example, $\sqrt{x} = -3$ has no real solution because a square root cannot equal a negative number. But if you square both sides without thinking, you get $x = 9$, which looks like an answer. Substituting back reveals the problem:

$$\sqrt{9} = 3 \neq -3$$

So $x = 9$ is an extraneous root and must be rejected. This is why checking is mandatory.

What an Extraneous Root Looks Like

An extraneous root is a value that:

- Comes out of your algebra correctly

- Does not satisfy the original equation when you substitute it back

When a root is extraneous, you write: *No solution* or state that the root is rejected.

Equations with the Radical Not Yet Isolated

Sometimes the radical is not already alone on one side. You must move any extra terms before squaring. For example:

$$\sqrt{x+2}+3=7$$

First subtract 3 from both sides to isolate the radical:

$$\sqrt{x+2}=4$$

Now square both sides:

$$x+2=16$$

Solve:

$$x=14$$

Check: $\sqrt{14+2}+3=\sqrt{16}+3=4+3=7$ ✓

Pro tip

- Always isolate the radical **before** squaring both sides. Squaring before isolating creates extra terms that are much harder to handle.
- Checking is not just a good habit — it is a **required step**. Extraneous roots are common in this topic, and only checking will catch them.
- When you square a binomial like $(x+3)^2$, expand it fully: $(x+3)^2=x^2+6x+9$. Do not just write x^2+9.
- If your check gives a negative value under a square root or a false equation, the root is extraneous and you must reject it.

Examples

Example A

Solve $\sqrt{x}=7$.

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$\left(\sqrt{x}\right)^2=7^2$$

$$x=49$$

Step 3: Check by substituting $x=49$ into the original equation.

$$\sqrt{49}=7\checkmark$$

The solution is $x = 49$.

Example B
Solve $\sqrt{x+5} = 4$.

Step 1: The radical is already isolated.
Step 2: Square both sides.

$$\left(\sqrt{x+5}\right)^2 = 4^2$$

$$x + 5 = 16$$

Step 3: Solve for x.

$$x = 16 - 5 = 11$$

Step 4: Check by substituting $x = 11$.

$$\sqrt{11+5} = \sqrt{16} = 4\checkmark$$

The solution is $x = 11$.

Example C
Solve $\sqrt{3x-2} + 1 = 6$.

Step 1: Isolate the radical. Subtract 1 from both sides.

$$\sqrt{3x-2} = 5$$

Step 2: Square both sides.

$$\left(\sqrt{3x-2}\right)^2 = 5^2$$

$$3x - 2 = 25$$

Step 3: Solve for x.

$$3x = 25 + 2 = 27$$

$$x = 9$$

Step 4: Check by substituting $x = 9$.

$$\sqrt{3(9)-2} + 1 = \sqrt{27-2} + 1 = \sqrt{25} + 1 = 5 + 1 = 6\checkmark$$

The solution is $x = 9$.

Example D — Extraneous Root
Solve $\sqrt{x+4} = -2$.

Step 1: The radical is already isolated.
Step 2: Square both sides.

$$x + 4 = 4$$

$$x = 0$$

Step 3: Check by substituting $x = 0$.

$$\sqrt{0+4} = \sqrt{4} = 2 \neq -2$$

The check fails. $x = 0$ is an extraneous root.

The original equation has **no solution** because a square root cannot equal a negative number.

Example E

Solve $\sqrt{2x+3} = x$.

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$2x + 3 = x^2$$

Step 3: Rearrange into standard form.

$$x^2 - 2x - 3 = 0$$

Step 4: Factor the quadratic.

$$(x-3)(x+1) = 0$$

Step 5: Solve for both values of x.

$$x = 3 \qquad \text{or} \qquad x = -1$$

Step 6: Check $x = 3$.

$$\sqrt{2(3)+3} = \sqrt{9} = 3\checkmark$$

Step 7: Check $x = -1$.

$$\sqrt{2(-1)+3} = \sqrt{1} = 1 \neq -1$$

$x = -1$ is an extraneous root and must be rejected.

The solution is $x = 3$.

Common mistakes

- **Squaring before isolating.** Always move all other terms away from the radical before squaring. Squaring a sum like $(\sqrt{x}+3)^2$ gives $x + 6\sqrt{x} + 9$, which is much harder to work with.
- **Skipping the check.** Extraneous roots are very common. A solution that looks correct algebraically may not work in the original equation.
- **Squaring a binomial incorrectly.** $(x+2)^2 = x^2+4x+4$, not x^2+4. Always expand fully.
- **Rejecting valid negatives in the radicand.** The expression under the radical must be non-negative, but the value of x itself can be negative. Always check by substituting.

Worksheet

1. Solve each equation. Check your answer.

   (a) $\sqrt{x} = 9$

   (b) $\sqrt{x} = 12$

   (c) $\sqrt{x} = 3$

   (d) $\sqrt{x} = 0$

2. Solve each equation. Check your answer.

   (a) $\sqrt{x+1} = 3$

   (b) $\sqrt{x-4} = 5$

   (c) $\sqrt{2x+1} = 5$

   (d) $\sqrt{3x-5} = 4$

3. Isolate the radical first, then solve. Check your answer.

   (a) $\sqrt{x} + 2 = 6$

   (b) $\sqrt{x} - 3 = 1$

   (c) $\sqrt{x+2} + 4 = 9$

   (d) $\sqrt{2x-3} - 1 = 4$

4. Solve each equation. If a root is extraneous, state that there is no solution.

   (a) $\sqrt{x+3} = -4$

   (b) $\sqrt{x-1} = -1$

   (c) $\sqrt{4x+1} = 0$

5. Solve $\sqrt{x+6} = x$. Check both solutions and reject any extraneous roots.

6. Solve $\sqrt{3x+4} = x$. Check both solutions and reject any extraneous roots.

7. Solve $2\sqrt{x+1} = 8$.

8. Solve $3\sqrt{2x-5} = 9$.

9. **Challenge:** Solve $\sqrt{x+7} = x+1$. Check all solutions.

10. **Challenge:** Solve $\sqrt{5x-4} = \sqrt{3x+6}$.

Answer Key

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1.

(a) $\sqrt{x} = 9$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x = 9^2 = 81$$

Step 3: Check.

$$\sqrt{81} = 9\checkmark$$

$$\boxed{x = 81}$$

(b) $\sqrt{x} = 12$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x = 12^2 = 144$$

Step 3: Check.

$$\sqrt{144} = 12\checkmark$$

$$\boxed{x = 144}$$

(c) $\sqrt{x} = 3$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x = 3^2 = 9$$

Step 3: Check.

$$\sqrt{9} = 3\checkmark$$

$$\boxed{x = 9}$$

(d) $\sqrt{x} = 0$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x = 0^2 = 0$$

Step 3: Check.

$$\sqrt{0} = 0\checkmark$$

$$\boxed{x = 0}$$

2.

(a) $\sqrt{x+1} = 3$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x + 1 = 9$$

Step 3: Solve.

$$x = 8$$

Step 4: Check.

$$\sqrt{8+1} = \sqrt{9} = 3\checkmark$$

$$\boxed{x = 8}$$

(b) $\sqrt{x-4} = 5$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x - 4 = 25$$

Step 3: Solve.

$$x = 29$$

Step 4: Check.

$$\sqrt{29-4} = \sqrt{25} = 5\checkmark$$

$$\boxed{x = 29}$$

(c) $\sqrt{2x+1} = 5$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$2x + 1 = 25$$

Step 3: Solve.

$$2x = 24 \qquad \Rightarrow \qquad x = 12$$

Step 4: Check.

$$\sqrt{2(12)+1} = \sqrt{25} = 5\checkmark$$

$$\boxed{x = 12}$$

(d) $\sqrt{3x-5} = 4$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$3x - 5 = 16$$

Step 3: Solve.

$$3x = 21 \qquad \Rightarrow \qquad x = 7$$

Step 4: Check.

$$\sqrt{3(7)-5} = \sqrt{21-5} = \sqrt{16} = 4\checkmark$$

$$\boxed{x = 7}$$

3.

(a) $\sqrt{x} + 2 = 6$
Step 1: Isolate the radical. Subtract 2 from both sides.

$$\sqrt{x} = 4$$

Step 2: Square both sides.

$$x = 16$$

Step 3: Check.

$$\sqrt{16} + 2 = 4 + 2 = 6\checkmark$$

$$\boxed{x = 16}$$

(b) $\sqrt{x} - 3 = 1$
Step 1: Isolate the radical. Add 3 to both sides.

$$\sqrt{x} = 4$$

Step 2: Square both sides.

$$x = 16$$

Step 3: Check.

$$\sqrt{16} - 3 = 4 - 3 = 1\checkmark$$

$$\boxed{x = 16}$$

(c) $\sqrt{x+2} + 4 = 9$
Step 1: Isolate the radical. Subtract 4 from both sides.

$$\sqrt{x+2} = 5$$

Step 2: Square both sides.

$$x + 2 = 25$$

Step 3: Solve.

$$x = 23$$

Step 4: Check.

$$\sqrt{23+2} + 4 = \sqrt{25} + 4 = 5 + 4 = 9\checkmark$$

$$\boxed{x = 23}$$

(d) $\sqrt{2x-3} - 1 = 4$
Step 1: Isolate the radical. Add 1 to both sides.

$$\sqrt{2x-3} = 5$$

Step 2: Square both sides.

$$2x - 3 = 25$$

Step 3: Solve.

$$2x = 28 \qquad \Rightarrow \qquad x = 14$$

Step 4: Check.

$$\sqrt{2(14) - 3} - 1 = \sqrt{25} - 1 = 5 - 1 = 4\checkmark$$

$$\boxed{x = 14}$$

4.

(a) $\sqrt{x + 3} = -4$

Step 1: A square root cannot equal a negative number. The left side is always greater than or equal to zero, but the right side is -4.

Step 2: There is no need to proceed. No real value of x will make a square root equal to -4.

$$\boxed{\text{No solution}}$$

(b) $\sqrt{x - 1} = -1$

Step 1: A square root cannot equal a negative number.

$$\boxed{\text{No solution}}$$

(c) $\sqrt{4x + 1} = 0$

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$4x + 1 = 0$$

Step 3: Solve.

$$4x = -1 \qquad \Rightarrow \qquad x = -\frac{1}{4}$$

Step 4: Check.

$$\sqrt{4\left(-\frac{1}{4}\right) + 1} = \sqrt{-1 + 1} = \sqrt{0} = 0\checkmark$$

$$\boxed{x = -\frac{1}{4}}$$

5. Solve $\sqrt{x + 6} = x$.

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$x + 6 = x^2$$

Step 3: Rearrange into standard form.

$$x^2 - x - 6 = 0$$

Step 4: Factor the quadratic.

$$(x-3)(x+2)=0$$

Step 5: Solve.

$$x=3 \qquad \text{or} \qquad x=-2$$

Step 6: Check $x=3$.

$$\sqrt{3+6}=\sqrt{9}=3\checkmark$$

Step 7: Check $x=-2$.

$$\sqrt{-2+6}=\sqrt{4}=2\neq -2$$

$x=-2$ is an extraneous root and must be rejected.

$$\boxed{x=3}$$

6. Solve $\sqrt{3x+4}=x$.

Step 1: The radical is already isolated.

Step 2: Square both sides.

$$3x+4=x^2$$

Step 3: Rearrange into standard form.

$$x^2-3x-4=0$$

Step 4: Factor the quadratic.

$$(x-4)(x+1)=0$$

Step 5: Solve.

$$x=4 \qquad \text{or} \qquad x=-1$$

Step 6: Check $x=4$.

$$\sqrt{3(4)+4}=\sqrt{16}=4\checkmark$$

Step 7: Check $x=-1$.

$$\sqrt{3(-1)+4}=\sqrt{1}=1\neq -1$$

$x=-1$ is an extraneous root and must be rejected.

$$\boxed{x=4}$$

7. Solve $2\sqrt{x+1}=8$.

Step 1: Isolate the radical. Divide both sides by 2.

$$\sqrt{x+1}=4$$

Step 2: Square both sides.

$$x+1=16$$

Step 3: Solve.

$$x = 15$$

Step 4: Check.

$$2\sqrt{15+1} = 2\sqrt{16} = 2 \times 4 = 8\checkmark$$

$$\boxed{x = 15}$$

8. Solve $3\sqrt{2x-5} = 9$.
Step 1: Isolate the radical. Divide both sides by 3.

$$\sqrt{2x-5} = 3$$

Step 2: Square both sides.

$$2x - 5 = 9$$

Step 3: Solve.

$$2x = 14 \qquad \Rightarrow \qquad x = 7$$

Step 4: Check.

$$3\sqrt{2(7)-5} = 3\sqrt{9} = 3 \times 3 = 9\checkmark$$

$$\boxed{x = 7}$$

9. Solve $\sqrt{x+7} = x + 1$.
Step 1: The radical is already isolated.
Step 2: Square both sides. Note that the right side is a binomial, so expand it carefully.

$$x + 7 = (x+1)^2$$

$$x + 7 = x^2 + 2x + 1$$

Step 3: Rearrange into standard form.

$$0 = x^2 + 2x + 1 - x - 7$$

$$0 = x^2 + x - 6$$

Step 4: Factor.

$$(x+3)(x-2) = 0$$

Step 5: Solve.

$$x = -3 \qquad \text{or} \qquad x = 2$$

Step 6: Check $x = 2$.

$$\sqrt{2+7} = \sqrt{9} = 3 \qquad \text{and} \qquad 2 + 1 = 3\checkmark$$

Step 7: Check $x = -3$.

$$\sqrt{-3+7} = \sqrt{4} = 2 \qquad \text{and} \qquad -3 + 1 = -2$$

$$2 \neq -2$$

$x = -3$ is an extraneous root and must be rejected.

$$\boxed{x = 2}$$

10. Solve $\sqrt{5x - 4} = \sqrt{3x + 6}$.
Step 1: Both sides are already isolated radicals.
Step 2: Square both sides.

$$5x - 4 = 3x + 6$$

Step 3: Solve by collecting like terms.

$$5x - 3x = 6 + 4$$

$$2x = 10$$

$$x = 5$$

Step 4: Check.

$$\sqrt{5(5) - 4} = \sqrt{21} \qquad \text{and} \qquad \sqrt{3(5) + 6} = \sqrt{21}\checkmark$$

$$\boxed{x = 5}$$

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