Unit 1 Review — Radical Expressions and Equations
ApexMath

Lesson Summaries

Lesson 1 — What is a Radical?

A radical is a way of asking what number, multiplied by itself a certain number of times, gives a value. The number inside is the **radicand**. The small number in the notch is the **index**. A square root has index 2; a cube root has index 3.

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Perfect squares to know: $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225$
Perfect cubes to know: $1, 8, 27, 64, 125, 216, 343$

Lesson 2 — Simplifying Radicals

Find the largest perfect square factor of the radicand, split the radical using the product rule, and evaluate the perfect square root.

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

A radical is fully simplified when the radicand has no perfect square factors other than 1.
Check: if $\sqrt{n} = a\sqrt{b}$, then $a^2 \times b$ must equal n.

Lesson 3 — Adding and Subtracting Radicals

Only **like radicals** (same radicand) can be combined. Always simplify every radical first, then add or subtract the coefficients. The radical part never changes.

$$a\sqrt{n} + b\sqrt{n} = (a+b)\sqrt{n}$$

Unlike radicals cannot be combined and must be left as separate terms.

Lesson 4 — Multiplying Radicals

Multiply coefficients together and radicands together. Simplify the result.

$$a\sqrt{m} \times b\sqrt{n} = ab\sqrt{mn}$$

A radical multiplied by itself equals the radicand: $\sqrt{n} \times \sqrt{n} = n$
For binomials, distribute every term and collect like terms at the end.

Lesson 5 — Dividing Radicals and Rationalizing the Denominator

Divide radicands directly when possible. A radical in the denominator must be removed by rationalizing.

- Simple radical denominator: multiply numerator and denominator by that radical.

- Binomial denominator: multiply by the conjugate. The conjugate of $(a+\sqrt{b})$ is $(a-\sqrt{b})$.

$$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$$

Lesson 6 — Solving Radical Equations

Isolate the radical, then square both sides (or cube both sides for a cube root). Solve the resulting equation. **Always check every solution** in the original equation.

Extraneous roots satisfy the squared equation but not the original. If a check fails, reject that root.

Lesson 7 — Extraneous Roots and Domain Restrictions

The domain of a square root expression requires the radicand to be ≥ 0.

$$\sqrt{f(x)} \text{ requires } f(x) \geq 0$$

If the denominator contains a radical, the radicand must be > 0.

Cube roots have no domain restriction — they are defined for all real numbers.

Extraneous roots appear because squaring removes sign information. A value outside the domain is always extraneous. A value inside the domain may still be extraneous — always check.

Review Practice

Section A — Evaluating and Simplifying

1. Evaluate each expression.

   (a) $\sqrt{169}$

   (b) $\sqrt[3]{125}$

   (c) $\sqrt[4]{81}$

   (d) $\sqrt{0}$

2. Simplify each radical completely.

   (a) $\sqrt{72}$

   (b) $\sqrt{108}$

   (c) $\sqrt{245}$

   (d) $3\sqrt{80}$

3. State the domain of each expression.

   (a) $\sqrt{5x - 10}$

   (b) $\sqrt{7 - x}$

   (c) $\dfrac{1}{\sqrt{x + 4}}$

   (d) $\sqrt[3]{x - 9}$

Section B — Operations with Radicals

4. Add or subtract. Simplify all radicals first.

(a) $\sqrt{32} + \sqrt{8}$

(b) $3\sqrt{12} - \sqrt{48}$

(c) $\sqrt{50} + \sqrt{18} - \sqrt{8}$

(d) $2\sqrt{45} - 3\sqrt{20} + \sqrt{5}$

5. Multiply and simplify.

   (a) $\sqrt{6} \times \sqrt{6}$

   (b) $3\sqrt{5} \times 4\sqrt{5}$

   (c) $2\sqrt{3} \times 5\sqrt{27}$

   (d) $\sqrt{8} \times \sqrt{18}$

6. Expand and simplify.

   (a) $\sqrt{5}(3 - \sqrt{5})$

(b) $(2+\sqrt{3})(2-\sqrt{3})$

(c) $(1+\sqrt{6})^2$

(d) $(3+\sqrt{2})(4-\sqrt{2})$

7. Rationalize the denominator and simplify.

(a) $\dfrac{8}{\sqrt{2}}$

(b) $\dfrac{3}{\sqrt{5}}$

(c) $\dfrac{4}{1+\sqrt{3}}$

(d) $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}$

Section C — Solving Equations

8. Solve each equation. Check your answers.

   (a) $\sqrt{x} = 11$

   (b) $\sqrt{x-3} = 6$

   (c) $\sqrt{2x+7} - 3 = 4$

   (d) $3\sqrt{x+1} = 12$

9. Solve each equation. Identify and reject any extraneous roots.

   (a) $\sqrt{x+5} = x - 1$

   (b) $\sqrt{2x+3} = x - 1$

10. Solve each equation.

   (a) $\sqrt[3]{4x-3} = 5$

   (b) $\sqrt[3]{2x+6} = -4$

Section D — Challenge Problems

11. Simplify $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{3}}$.

12. Solve $\sqrt{x+10}+\sqrt{x}=5$.

13. A rectangle has an area of $\sqrt{300}$ cm^2 and a width of $\sqrt{3}$ cm. Find the length in simplest radical form.

Answer Key

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Section A — Evaluating and Simplifying

1.

(a) $\sqrt{169}$
Step 1: $13 \times 13 = 169$

$$\boxed{13}$$

(b) $\sqrt[3]{125}$
Step 1: $5 \times 5 \times 5 = 125$

$$\boxed{5}$$

(c) $\sqrt[4]{81}$
Step 1: $3 \times 3 \times 3 \times 3 = 81$

$$\boxed{3}$$

(d) $\sqrt{0}$
Step 1: $0 \times 0 = 0$

$$\boxed{0}$$

2.

(a) Simplify $\sqrt{72}$.
Step 1: Largest perfect square factor of 72 is 36.

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

(b) Simplify $\sqrt{108}$.
Step 1: Largest perfect square factor of 108 is 36.

$$\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$$

$$\boxed{6\sqrt{3}}$$

(c) Simplify $\sqrt{245}$.
Step 1: Largest perfect square factor of 245 is 49.

$$\sqrt{245} = \sqrt{49 \times 5} = 7\sqrt{5}$$

$$\boxed{7\sqrt{5}}$$

(d) Simplify $3\sqrt{80}$.
Step 1: Largest perfect square factor of 80 is 16.

$$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$$

Step 2: Multiply by the coefficient.

$$3 \times 4\sqrt{5} = 12\sqrt{5}$$

$$\boxed{12\sqrt{5}}$$

3.

(a) Domain of $\sqrt{5x - 10}$.
Step 1: Set radicand ≥ 0.

$$5x - 10 \geq 0 \quad \Rightarrow \quad 5x \geq 10 \quad \Rightarrow \quad x \geq 2$$

$$\boxed{x \geq 2}$$

(b) Domain of $\sqrt{7 - x}$.
Step 1: Set radicand ≥ 0.

$$7 - x \geq 0 \quad \Rightarrow \quad -x \geq -7 \quad \Rightarrow \quad x \leq 7$$

$$\boxed{x \leq 7}$$

(c) Domain of $\dfrac{1}{\sqrt{x+4}}$.
Step 1: Denominator cannot be zero, so radicand must be > 0.

$$x + 4 > 0 \quad \Rightarrow \quad x > -4$$

$$\boxed{x > -4}$$

(d) Domain of $\sqrt[3]{x - 9}$.
Step 1: Cube roots are defined for all real numbers.

$$\boxed{\text{All real numbers}}$$

Section B — Operations with Radicals

4.

(a) $\sqrt{32} + \sqrt{8}$
Step 1: Simplify $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$
Step 2: Simplify $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

Step 3: Add.

$$4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

(b) $3\sqrt{12} - \sqrt{48}$
Step 1: Simplify $3\sqrt{12}$: $\sqrt{12} = 2\sqrt{3}$, so $3\sqrt{12} = 6\sqrt{3}$
Step 2: Simplify $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$
Step 3: Subtract.

$$6\sqrt{3} - 4\sqrt{3} = 2\sqrt{3}$$

$$\boxed{2\sqrt{3}}$$

(c) $\sqrt{50} + \sqrt{18} - \sqrt{8}$
Step 1: $\sqrt{50} = 5\sqrt{2}$, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{8} = 2\sqrt{2}$
Step 2: Combine.

$$5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

(d) $2\sqrt{45} - 3\sqrt{20} + \sqrt{5}$
Step 1: Simplify $2\sqrt{45}$: $\sqrt{45} = 3\sqrt{5}$, so $2\sqrt{45} = 6\sqrt{5}$
Step 2: Simplify $3\sqrt{20}$: $\sqrt{20} = 2\sqrt{5}$, so $3\sqrt{20} = 6\sqrt{5}$
Step 3: $\sqrt{5}$ stays as $1\sqrt{5}$
Step 4: Combine.

$$6\sqrt{5} - 6\sqrt{5} + \sqrt{5} = \sqrt{5}$$

$$\boxed{\sqrt{5}}$$

5.

(a) $\sqrt{6} \times \sqrt{6}$
Step 1: A radical times itself equals the radicand.

$$\sqrt{6} \times \sqrt{6} = 6$$

$$\boxed{6}$$

(b) $3\sqrt{5} \times 4\sqrt{5}$
Step 1: Multiply coefficients: $3 \times 4 = 12$
Step 2: Multiply radicands: $\sqrt{5} \times \sqrt{5} = 5$
Step 3: $12 \times 5 = 60$

$$\boxed{60}$$

(c) $2\sqrt{3} \times 5\sqrt{27}$
Step 1: Multiply coefficients: $2 \times 5 = 10$
Step 2: Multiply radicands: $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$

Step 3: $10 \times 9 = 90$

$$\boxed{90}$$

(d) $\sqrt{8} \times \sqrt{18}$
Step 1: Multiply radicands: $\sqrt{8 \times 18} = \sqrt{144} = 12$

$$\boxed{12}$$

6.

(a) $\sqrt{5}(3 - \sqrt{5})$
Step 1: Distribute.

$$\sqrt{5} \times 3 - \sqrt{5} \times \sqrt{5} = 3\sqrt{5} - 5$$

$$\boxed{3\sqrt{5} - 5}$$

(b) $(2 + \sqrt{3})(2 - \sqrt{3})$
Step 1: Difference of squares pattern.

$$2^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

$$\boxed{1}$$

(c) $(1 + \sqrt{6})^2$
Step 1: Expand as $(1 + \sqrt{6})(1 + \sqrt{6})$.

$$1 \times 1 + 1 \times \sqrt{6} + \sqrt{6} \times 1 + \sqrt{6} \times \sqrt{6}$$

$$= 1 + \sqrt{6} + \sqrt{6} + 6 = 7 + 2\sqrt{6}$$

$$\boxed{7 + 2\sqrt{6}}$$

(d) $(3 + \sqrt{2})(4 - \sqrt{2})$
Step 1: Expand.

$$3 \times 4 + 3 \times (-\sqrt{2}) + \sqrt{2} \times 4 + \sqrt{2} \times (-\sqrt{2})$$

$$= 12 - 3\sqrt{2} + 4\sqrt{2} - 2$$

Step 2: Collect terms.

$$(12 - 2) + (-3\sqrt{2} + 4\sqrt{2}) = 10 + \sqrt{2}$$

$$\boxed{10 + \sqrt{2}}$$

7.

(a) $\dfrac{8}{\sqrt{2}}$

Step 1: Multiply numerator and denominator by $\sqrt{2}$.

$$\frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$\boxed{4\sqrt{2}}$$

(b) $\dfrac{3}{\sqrt{5}}$

Step 1: Multiply by $\dfrac{\sqrt{5}}{\sqrt{5}}$.

$$\frac{3\sqrt{5}}{5}$$

$$\boxed{\frac{3\sqrt{5}}{5}}$$

(c) $\dfrac{4}{1+\sqrt{3}}$

Step 1: Conjugate of $(1+\sqrt{3})$ is $(1-\sqrt{3})$.

Step 2: Multiply.

$$\frac{4}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

Step 3: Expand denominator.

$$(1+\sqrt{3})(1-\sqrt{3}) = 1 - 3 = -2$$

Step 4: Expand numerator.

$$4(1-\sqrt{3}) = 4 - 4\sqrt{3}$$

Step 5: Write and simplify.

$$\frac{4-4\sqrt{3}}{-2} = -2 + 2\sqrt{3}$$

$$\boxed{2\sqrt{3} - 2}$$

(d) $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}$

Step 1: Conjugate of $(2-\sqrt{5})$ is $(2+\sqrt{5})$.

Step 2: Expand denominator.

$$(2-\sqrt{5})(2+\sqrt{5}) = 4 - 5 = -1$$

Step 3: Expand numerator.

$$(2+\sqrt{5})(2+\sqrt{5}) = 4 + 2\sqrt{5} + 2\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

Step 4: Divide by -1.

$$\frac{9+4\sqrt{5}}{-1} = -9 - 4\sqrt{5}$$

$$\boxed{-9 - 4\sqrt{5}}$$

Section C — Solving Equations

8.

(a) $\sqrt{x} = 11$
Step 1: Square both sides.

$$x = 121$$

Step 2: Check: $\sqrt{121} = 11$ ✓

$$\boxed{x = 121}$$

(b) $\sqrt{x-3} = 6$
Step 1: Square both sides.

$$x - 3 = 36 \quad \Rightarrow \quad x = 39$$

Step 2: Check: $\sqrt{39-3} = \sqrt{36} = 6$ ✓

$$\boxed{x = 39}$$

(c) $\sqrt{2x+7} - 3 = 4$
Step 1: Isolate the radical.

$$\sqrt{2x+7} = 7$$

Step 2: Square both sides.

$$2x + 7 = 49 \quad \Rightarrow \quad 2x = 42 \quad \Rightarrow \quad x = 21$$

Step 3: Check: $\sqrt{2(21)+7} - 3 = \sqrt{49} - 3 = 7 - 3 = 4$ ✓

$$\boxed{x = 21}$$

(d) $3\sqrt{x+1} = 12$
Step 1: Isolate the radical. Divide both sides by 3.

$$\sqrt{x+1} = 4$$

Step 2: Square both sides.

$$x + 1 = 16 \quad \Rightarrow \quad x = 15$$

Step 3: Check: $3\sqrt{15+1} = 3\sqrt{16} = 3 \times 4 = 12$ ✓

$$\boxed{x = 15}$$

9.

(a) $\sqrt{x+5} = x - 1$
Step 1: Domain: $x + 5 \geq 0$ so $x \geq -5$. Also $x - 1 \geq 0$ so $x \geq 1$.
Step 2: Square both sides.

$$x + 5 = (x-1)^2 = x^2 - 2x + 1$$

Step 3: Rearrange.

$$0 = x^2 - 2x + 1 - x - 5 = x^2 - 3x - 4$$

Step 4: Factor.

$$(x-4)(x+1) = 0 \quad \Rightarrow \quad x = 4 \quad \text{or} \quad x = -1$$

Step 5: Check $x = 4$.

$$\sqrt{4+5} = \sqrt{9} = 3 \qquad \text{and} \qquad 4 - 1 = 3\checkmark$$

Step 6: Check $x = -1$.

$$\sqrt{-1+5} = \sqrt{4} = 2 \qquad \text{and} \qquad -1 - 1 = -2$$

$2 \neq -2$, so $x = -1$ is extraneous.

$$\boxed{x = 4}$$

(b) $\sqrt{2x+3} = x - 1$
Step 1: Domain: $2x + 3 \geq 0$ so $x \geq -\dfrac{3}{2}$. Also $x - 1 \geq 0$ so $x \geq 1$.
Step 2: Square both sides.

$$2x + 3 = (x-1)^2 = x^2 - 2x + 1$$

Step 3: Rearrange.

$$0 = x^2 - 2x + 1 - 2x - 3 = x^2 - 4x - 2$$

Step 4: Use the quadratic formula since this does not factor neatly.

$$x = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

Step 5: The two solutions are $x = 2 + \sqrt{6}$ and $x = 2 - \sqrt{6}$.
Note: $\sqrt{6} \approx 2.449$, so $2 - \sqrt{6} \approx -0.449 < 1$. This violates $x \geq 1$.

Step 6: Check $x = 2 + \sqrt{6}$.

$$2x + 3 = 2(2 + \sqrt{6}) + 3 = 7 + 2\sqrt{6}$$

$$(x - 1)^2 = (1 + \sqrt{6})^2 = 1 + 2\sqrt{6} + 6 = 7 + 2\sqrt{6} \checkmark$$

Step 7: $x = 2 - \sqrt{6}$ is extraneous.

$$\boxed{x = 2 + \sqrt{6}}$$

10.

(a) $\sqrt[3]{4x - 3} = 5$
Step 1: Cube both sides.

$$4x - 3 = 125$$

Step 2: Solve.

$$4x = 128 \quad \Rightarrow \quad x = 32$$

Step 3: Check: $\sqrt[3]{4(32) - 3} = \sqrt[3]{125} = 5$ ✓

$$\boxed{x = 32}$$

(b) $\sqrt[3]{2x + 6} = -4$
Step 1: Cube both sides.

$$2x + 6 = (-4)^3 = -64$$

Step 2: Solve.

$$2x = -70 \quad \Rightarrow \quad x = -35$$

Step 3: Check: $\sqrt[3]{2(-35) + 6} = \sqrt[3]{-64} = -4$ ✓

$$\boxed{x = -35}$$

Section D — Challenge Problems

11. Simplify $\dfrac{\sqrt{6} + \sqrt{3}}{\sqrt{3}}$.
Step 1: Split into two separate fractions.

$$\frac{\sqrt{6}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{3}}$$

Step 2: Simplify each fraction.

$$\sqrt{\frac{6}{3}} + 1 = \sqrt{2} + 1$$

$$\boxed{\sqrt{2} + 1}$$

12. Solve $\sqrt{x+10}+\sqrt{x}=5$.
Step 1: Isolate one radical.

$$\sqrt{x+10}=5-\sqrt{x}$$

Step 2: Square both sides.

$$x+10=(5-\sqrt{x})^2=25-10\sqrt{x}+x$$

Step 3: Subtract x from both sides.

$$10=25-10\sqrt{x}$$

Step 4: Isolate the remaining radical.

$$10\sqrt{x}=15 \quad \Rightarrow \quad \sqrt{x}=\frac{3}{2}$$

Step 5: Square both sides.

$$x=\frac{9}{4}$$

Step 6: Check.

$$\sqrt{\frac{9}{4}+10}+\sqrt{\frac{9}{4}}=\sqrt{\frac{49}{4}}+\frac{3}{2}=\frac{7}{2}+\frac{3}{2}=\frac{10}{2}=5\checkmark$$

$$\boxed{x=\frac{9}{4}}$$

13. Rectangle with area $\sqrt{300}$ cm^2 and width $\sqrt{3}$ cm. Find the length.
Step 1: Recall the area formula.

$$A=l\times w \quad \Rightarrow \quad l=\frac{A}{w}$$

Step 2: Substitute.

$$l=\frac{\sqrt{300}}{\sqrt{3}}$$

Step 3: Divide the radicands.

$$l=\sqrt{\frac{300}{3}}=\sqrt{100}=10$$

Step 4: Write the answer with units.

$$\boxed{l=10\text{ cm}}$$

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End of Unit 1 Review • ApexMath
