What is a Radical?
ApexMath

Notes

The Big Idea

A **radical** is a way of asking: "What number, multiplied by itself a certain number of times, gives me this value?"

The most common radical you will see is the **square root**. The square root asks: "What number times itself equals this?"

For example:

$$\sqrt{9} = 3 \quad \text{because} \quad 3 \times 3 = 9$$

Parts of a Radical

Every radical has two main parts:

- The **radical symbol**: $\sqrt{}$
- The **radicand**: the number or expression sitting *inside* the radical symbol

For example, in $\sqrt{25}$, the radicand is 25.

The Index

Radicals can also have an **index**, which is the small number written in the notch of the radical symbol. The index tells you how many times a number must be multiplied by itself.

$$\sqrt[n]{a}$$

- When the index is 2, it is called a **square root**. We usually leave the 2 out and just write $\sqrt{}$.
- When the index is 3, it is called a **cube root**: $\sqrt[3]{}$
- When the index is 4, it is called a **fourth root**: $\sqrt[4]{}$

Examples:

$$\sqrt{16} = 4 \quad \text{because} \quad 4 \times 4 = 16$$

$$\sqrt[3]{8} = 2 \quad \text{because} \quad 2 \times 2 \times 2 = 8$$

$$\sqrt[4]{81} = 3 \quad \text{because} \quad 3 \times 3 \times 3 \times 3 = 81$$

Perfect Squares

A **perfect square** is any number that is the result of multiplying a whole number by itself. Recognizing perfect squares will make your life much easier throughout this unit.

$$1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100,\ 121,\ 144,\ 169,\ 196,\ 225$$

For example, 36 is a perfect square because $6 \times 6 = 36$, so $\sqrt{36} = 6$.

Perfect Cubes

A **perfect cube** is any number that is the result of multiplying a whole number by itself three times.

$$1,\ 8,\ 27,\ 64,\ 125,\ 216,\ 343$$

For example, 27 is a perfect cube because $3 \times 3 \times 3 = 27$, so $\sqrt[3]{27} = 3$.

Radicals and Exponents — They Are Connected

A radical can always be rewritten as a fractional exponent. This connection becomes very useful later in the course.

$$\sqrt{a} = a^{\frac{1}{2}} \qquad \sqrt[3]{a} = a^{\frac{1}{3}} \qquad \sqrt[n]{a} = a^{\frac{1}{n}}$$

You do not need to memorize this right now, but keep it in the back of your mind.

Pro tip

- Memorize perfect squares from 1 to 225 and perfect cubes from 1 to 343. You will use them constantly. Write them on a sticky note and keep it on your desk.
- If you are ever unsure whether a number is a perfect square, try multiplying whole numbers by themselves until you reach it. If you overshoot without hitting it exactly, it is not a perfect square.
- The square root of a number always gives you a **positive** answer in this course unless stated otherwise. For example, $\sqrt{49} = 7$, not -7.
- Do not confuse the **index** with the **radicand**. The index lives in the notch on top; the radicand lives underneath the bar.

Examples

Example A

Evaluate $\sqrt{144}$.

Step 1: Ask yourself what perfect square gives 144.

$$12 \times 12 = 144$$

Step 2: Write the answer.

$$\sqrt{144} = 12$$

Example B

Evaluate $\sqrt[3]{64}$.

Step 1: Ask yourself what number multiplied by itself three times gives 64.

$$4 \times 4 \times 4 = 64$$

Step 2: Write the answer.

$$\sqrt[3]{64} = 4$$

Example C
Is $\sqrt{50}$ a whole number? Explain.

Step 1: Check whether 50 is a perfect square by finding the perfect squares closest to it.

$$7 \times 7 = 49 \qquad 8 \times 8 = 64$$

Step 2: Since 50 falls between 49 and 64, it is **not** a perfect square.
Step 3: Conclude.
$\sqrt{50}$ is **not** a whole number. It is a value that falls between 7 and 8.

Example D
Rewrite $\sqrt[4]{x}$ using exponent notation.

Step 1: Identify the index. The index is 4.
Step 2: Apply the rule $\sqrt[n]{a} = a^{\frac{1}{n}}$.

$$\sqrt[4]{x} = x^{\frac{1}{4}}$$

Common mistakes

- **Forgetting that the square root gives a positive answer.** In this course, $\sqrt{36} = 6$, not ± 6. The $\pm$ only appears when you are solving an equation.
- **Confusing the index and the radicand.** In $\sqrt[3]{8}$, the 3 is the index (how many times), and 8 is what you are taking the root of.
- **Assuming all radicals are whole numbers.** Only perfect squares produce whole number square roots. $\sqrt{10}$ is not 3; it is slightly more than 3.
- **Skipping the check.** When evaluating a radical, always verify your answer by multiplying it back out.

Worksheet

1. Evaluate each of the following.

   (a) $\sqrt{49}$

   (b) $\sqrt{121}$

(c) $\sqrt{196}$

(d) $\sqrt{225}$

2. Evaluate each of the following cube roots.

(a) $\sqrt[3]{27}$

(b) $\sqrt[3]{125}$

(c) $\sqrt[3]{216}$

3. Determine whether each number is a perfect square. Write **yes** or **no**, and explain briefly.

(a) 81

(b) 50

(c) 169

(d) 200

4. List all perfect squares from 1 to 100.

5. Between which two consecutive whole numbers does each radical fall?

   (a) $\sqrt{20}$

   (b) $\sqrt{75}$

   (c) $\sqrt{150}$

6. Rewrite each radical using exponent notation.

   (a) $\sqrt{m}$

   (b) $\sqrt[3]{y}$

   (c) $\sqrt[5]{k}$

7. Evaluate $\sqrt[4]{16}$ and verify your answer by expanding.

8. A square has an area of 169 cm^2. What is the side length of the square? Show your work.

9. A cube has a volume of 512 cm^3. What is the side length of the cube? Show your work.

10. **Challenge:** Without a calculator, determine whether $\sqrt{400}$ is a whole number. If it is, state the value.

Answer Key

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1.

(a) Evaluate $\sqrt{49}$.
Step 1: Identify the radicand. The radicand is 49.
Step 2: Ask what number multiplied by itself gives 49.

$$7 \times 7 = 49$$

Step 3: Write the answer.

$$\boxed{7}$$

(b) Evaluate $\sqrt{121}$.
Step 1: The radicand is 121.
Step 2: Ask what number multiplied by itself gives 121.

$$11 \times 11 = 121$$

Step 3: Write the answer.

$$\boxed{11}$$

(c) Evaluate $\sqrt{196}$.
Step 1: The radicand is 196.
Step 2: Ask what number multiplied by itself gives 196.

$$14 \times 14 = 196$$

Step 3: Write the answer.

$$\boxed{14}$$

(d) Evaluate $\sqrt{225}$.
Step 1: The radicand is 225.
Step 2: Ask what number multiplied by itself gives 225.

$$15 \times 15 = 225$$

Step 3: Write the answer.

$$\boxed{15}$$

2.

(a) Evaluate $\sqrt[3]{27}$.
Step 1: The index is 3 and the radicand is 27.

Step 2: Ask what number multiplied by itself three times gives 27.

$$3 \times 3 \times 3 = 27$$

Step 3: Write the answer.

$$\boxed{3}$$

(b) Evaluate $\sqrt[3]{125}$.
Step 1: The index is 3 and the radicand is 125.
Step 2: Ask what number multiplied by itself three times gives 125.

$$5 \times 5 \times 5 = 125$$

Step 3: Write the answer.

$$\boxed{5}$$

(c) Evaluate $\sqrt[3]{216}$.
Step 1: The index is 3 and the radicand is 216.
Step 2: Ask what number multiplied by itself three times gives 216.

$$6 \times 6 \times 6 = 216$$

Step 3: Write the answer.

$$\boxed{6}$$

3.

(a) Is 81 a perfect square?
Step 1: Try multiplying whole numbers by themselves to reach 81.

$$9 \times 9 = 81$$

Step 2: A whole number times itself gives 81 exactly, so yes, it is a perfect square.

$$\boxed{\text{Yes.} \quad \sqrt{81} = 9}$$

(b) Is 50 a perfect square?
Step 1: Find the perfect squares on either side of 50.

$$7 \times 7 = 49 \qquad 8 \times 8 = 64$$

Step 2: 50 falls between 49 and 64. No whole number times itself gives 50 exactly.

$$\boxed{\text{No. 50 is not a perfect square.}}$$

(c) Is 169 a perfect square?

Step 1: Try multiplying whole numbers by themselves to reach 169.

$$13 \times 13 = 169$$

Step 2: A whole number times itself gives 169 exactly, so yes, it is a perfect square.

$$\boxed{\text{Yes.} \quad \sqrt{169} = 13}$$

(d) Is 200 a perfect square?
Step 1: Find the perfect squares on either side of 200.

$$14 \times 14 = 196 \qquad 15 \times 15 = 225$$

Step 2: 200 falls between 196 and 225. No whole number times itself gives 200 exactly.

$$\boxed{\text{No. 200 is not a perfect square.}}$$

4. List all perfect squares from 1 to 100.
Step 1: Multiply each whole number from 1 to 10 by itself.

$$1^2 = 1, \quad 2^2 = 4, \quad 3^2 = 9, \quad 4^2 = 16, \quad 5^2 = 25$$

$$6^2 = 36, \quad 7^2 = 49, \quad 8^2 = 64, \quad 9^2 = 81, \quad 10^2 = 100$$

Step 2: List them in order.

$$\boxed{1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100}$$

5.

(a) Between which two whole numbers does $\sqrt{20}$ fall?
Step 1: Find the perfect squares directly above and below 20.

$$4 \times 4 = 16 \qquad 5 \times 5 = 25$$

Step 2: Since $16 < 20 < 25$:

$$4 < \sqrt{20} < 5$$

$$\boxed{\sqrt{20} \text{ falls between 4 and 5}}$$

(b) Between which two whole numbers does $\sqrt{75}$ fall?
Step 1: Find the perfect squares directly above and below 75.

$$8 \times 8 = 64 \qquad 9 \times 9 = 81$$

Step 2: Since $64 < 75 < 81$:

$$8 < \sqrt{75} < 9$$

$$\boxed{\sqrt{75} \text{ falls between 8 and 9}}$$

(c) Between which two whole numbers does $\sqrt{150}$ fall?
Step 1: Find the perfect squares directly above and below 150.

$$12 \times 12 = 144 \qquad 13 \times 13 = 169$$

Step 2: Since $144 < 150 < 169$:

$$12 < \sqrt{150} < 13$$

$$\boxed{\sqrt{150} \text{ falls between 12 and 13}}$$

6.

(a) Rewrite $\sqrt{m}$ using exponent notation.
Step 1: The index is 2 (a square root). Apply the rule $\sqrt[n]{a} = a^{\frac{1}{n}}$.
Step 2: Replace n with 2.

$$\boxed{m^{\frac{1}{2}}}$$

(b) Rewrite $\sqrt[3]{y}$ using exponent notation.
Step 1: The index is 3. Apply the rule $\sqrt[n]{a} = a^{\frac{1}{n}}$.
Step 2: Replace n with 3.

$$\boxed{y^{\frac{1}{3}}}$$

(c) Rewrite $\sqrt[5]{k}$ using exponent notation.
Step 1: The index is 5. Apply the rule $\sqrt[n]{a} = a^{\frac{1}{n}}$.
Step 2: Replace n with 5.

$$\boxed{k^{\frac{1}{5}}}$$

7. Evaluate $\sqrt[4]{16}$ and verify your answer by expanding.
Step 1: The index is 4. Ask what number multiplied by itself four times gives 16.

$$2 \times 2 \times 2 \times 2 = 16$$

Step 2: Verify by writing it as a power.

$$2^4 = 16\checkmark$$

Step 3: Write the answer.

$$\boxed{\sqrt[4]{16} = 2}$$

8. A square has an area of 169 cm^2. Find the side length.

Step 1: Recall the formula for the area of a square.

$$A = s^2$$

Step 2: Substitute the known area.

$$169 = s^2$$

Step 3: Take the square root of both sides to isolate s.

$$s = \sqrt{169}$$

Step 4: Evaluate. Ask what number times itself gives 169.

$$13 \times 13 = 169$$

Step 5: Write the answer with units.

$$\boxed{s = 13 \text{ cm}}$$

9. A cube has a volume of 512 cm^3. Find the side length.

Step 1: Recall the formula for the volume of a cube.

$$V = s^3$$

Step 2: Substitute the known volume.

$$512 = s^3$$

Step 3: Take the cube root of both sides to isolate s.

$$s = \sqrt[3]{512}$$

Step 4: Ask what number multiplied by itself three times gives 512.

$$8 \times 8 \times 8 = 512$$

Step 5: Write the answer with units.

$$\boxed{s = 8 \text{ cm}}$$

10. Is $\sqrt{400}$ a whole number?

Step 1: Ask whether 400 is a perfect square. Try 20.

$$20 \times 20 = 400$$

Step 2: Yes, 400 is a perfect square because a whole number times itself gives exactly 400.

Step 3: Write the answer.

$$\boxed{\sqrt{400} = 20}$$

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