Multiplying and Dividing Rational Expressions
ApexMath

Notes

The Big Idea

Multiplying and dividing rational expressions works exactly the same way as multiplying and dividing numerical fractions. The only extra steps are factoring first and stating non-permissible values.

Recall how numerical fractions multiply:

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Rational expressions follow the same rule:

$$\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$$

Multiplying Rational Expressions — The Method

- Step 1: Factor every numerator and denominator completely.
- Step 2: State the non-permissible values from all denominators before cancelling anything.
- Step 3: Multiply numerators together and denominators together.
- Step 4: Cancel any common factors that appear in both the numerator and denominator.
- Step 5: Write the simplified result.

In practice, it is more efficient to cancel common factors *before* multiplying across. This keeps numbers smaller and avoids large expressions.

Example of Multiplying

Simplify $\dfrac{x+2}{x-3} \times \dfrac{x^2-9}{x^2-4}$.

Step 1: Factor all numerators and denominators.

$$x^2 - 9 = (x-3)(x+3) \qquad x^2 - 4 = (x-2)(x+2)$$

Step 2: Rewrite.

$$\frac{(x+2)}{(x-3)} \times \frac{(x-3)(x+3)}{(x-2)(x+2)}$$

Step 3: State NPVs from all denominators: $x \neq 3$, $x \neq 2$, $x \neq -2$.
Step 4: Cancel common factors $(x+2)$ and $(x-3)$.

$$\frac{(x+2)}{(x-3)} \times \frac{(x-3)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x-2}$$

The answer is $\dfrac{x+3}{x-2}$, where $x \neq 3$, $x \neq 2$, and $x \neq -2$.

Dividing Rational Expressions

Dividing by a fraction is the same as multiplying by its reciprocal. This is the same rule you learned with numerical fractions:

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$$

For rational expressions:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$$

The method is:

- Step 1: Rewrite the division as multiplication by the reciprocal. Flip the second fraction.
- Step 2: Factor all numerators and denominators completely.
- Step 3: State the non-permissible values from **all** denominators — including the denominator of the flipped fraction, which was originally a numerator.
- Step 4: Cancel common factors and simplify.

Important Note on NPVs When Dividing

When you flip the second fraction, its numerator becomes a denominator. This means you must also check whether that expression can equal zero, and add those restrictions.

For example, in $\dfrac{A}{B} \div \dfrac{C}{D}$, the NPVs come from B, D, **and** C (since C becomes a denominator after flipping).

Pro tip

- Always factor **before** you multiply or cancel. Never try to cancel from an unfactored expression.
- For division, write out the reciprocal step explicitly. Skipping it is a common source of errors.
- Collect all non-permissible values from every denominator that appears at any point — including the one created when you flip during division.

- After cancelling, always check that no further simplification is possible in your result.
- Watch for the $\dfrac{x-a}{a-x} = -1$ pattern. If you see a numerator and denominator that are negatives of each other, they cancel to -1.

Examples

Example A

Simplify $\dfrac{3x}{x+4} \times \dfrac{x^2-16}{6x^2}$. State all NPVs.

Step 1: Factor all numerators and denominators.

$$x^2 - 16 = (x-4)(x+4) \qquad 6x^2 = 6 \cdot x \cdot x$$

Step 2: Rewrite.

$$\frac{3x}{(x+4)} \times \frac{(x-4)(x+4)}{6x^2}$$

Step 3: NPVs. Denominators are $(x+4)$ and $6x^2$.

$$x + 4 = 0 \Rightarrow x = -4 \qquad 6x^2 = 0 \Rightarrow x = 0$$

So $x \neq -4$ and $x \neq 0$.

Step 4: Cancel common factors. $(x+4)$ cancels, one x cancels, and 3 and 6 reduce.

$$\frac{3 \cdot x}{(x+4)} \times \frac{(x-4)(x+4)}{6 \cdot x \cdot x} = \frac{3(x-4)}{6x} = \frac{x-4}{2x}$$

The answer is $\dfrac{x-4}{2x}$, where $x \neq 0$ and $x \neq -4$.

Example B

Simplify $\dfrac{x^2-x-6}{x^2-4} \times \dfrac{x+2}{x-3}$. State all NPVs.

Step 1: Factor all numerators and denominators.

$$x^2 - x - 6 = (x-3)(x+2) \qquad x^2 - 4 = (x-2)(x+2)$$

Step 2: Rewrite.

$$\frac{(x-3)(x+2)}{(x-2)(x+2)} \times \frac{(x+2)}{(x-3)}$$

Step 3: NPVs from denominators $(x-2)(x+2)$ and $(x-3)$.

$$x \neq 2, \quad x \neq -2, \quad x \neq 3$$

Step 4: Cancel $(x-3)$ and one $(x+2)$.

$$\frac{(x-3)(x+2)}{(x-2)(x+2)} \times \frac{(x+2)}{(x-3)}$$

Wait — there are two $(x+2)$ factors in the numerator and two in the denominator. Cancel both.

$$= \frac{1}{x-2} \times \frac{(x+2)}{1}$$

Let us redo this carefully by writing all factors in one line:

$$\frac{(x-3)(x+2)(x+2)}{(x-2)(x+2)(x-3)} = \frac{x+2}{x-2}$$

The answer is $\dfrac{x+2}{x-2}$, where $x \neq 2$, $x \neq -2$, and $x \neq 3$.

Example C

Simplify $\dfrac{x^2-1}{x+3} \div \dfrac{x-1}{x+3}$. State all NPVs.

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2-1}{x+3} \times \frac{x+3}{x-1}$$

Step 2: Factor.

$$x^2 - 1 = (x-1)(x+1)$$

Step 3: NPVs. Denominators at any point are $(x+3)$ and $(x-1)$.

Also, the original second fraction had denominator $(x+3)$ and numerator $(x-1)$. After flipping, $(x-1)$ is now a denominator too.

$$x \neq -3 \quad \text{and} \quad x \neq 1$$

Step 4: Rewrite and cancel.

$$\frac{(x-1)(x+1)}{(x+3)} \times \frac{(x+3)}{(x-1)} = \frac{(x-1)(x+1)}{x-1} = x+1$$

The answer is $x+1$, where $x \neq -3$ and $x \neq 1$.

Example D

Simplify $\dfrac{2x^2+4x}{x^2-9} \div \dfrac{4x}{x-3}$. State all NPVs.

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{2x^2+4x}{x^2-9} \times \frac{x-3}{4x}$$

Step 2: Factor.

$$2x^2 + 4x = 2x(x+2) \qquad x^2 - 9 = (x-3)(x+3)$$

Step 3: NPVs from all denominators: $(x^2 - 9)$, $(4x)$, and the original numerator of the second fraction $(4x)$ becomes a denominator after flipping. Also the original denominator of the second fraction $(x - 3)$ was already a denominator.

$$x \neq 3, \quad x \neq -3, \quad x \neq 0$$

Step 4: Rewrite and cancel.

$$\frac{2x(x+2)}{(x-3)(x+3)} \times \frac{(x-3)}{4x} = \frac{2x(x+2)(x-3)}{(x+3)(x-3)\cdot 4x} = \frac{2(x+2)}{4(x+3)} = \frac{x+2}{2(x+3)}$$

The answer is $\dfrac{x+2}{2(x+3)}$, where $x \neq 3$, $x \neq -3$, and $x \neq 0$.

Common mistakes

- **Forgetting to flip the second fraction when dividing.** Division means multiply by the reciprocal. Always rewrite the division as multiplication first before doing anything else.
- **Missing NPVs from the flipped fraction.** When you flip the second fraction in a division, its original numerator becomes a denominator. Set it equal to zero and add that restriction.
- **Cancelling before factoring.** You can only cancel factors. Factor everything completely first, then look for common factors to cancel.
- **Cancelling across addition or subtraction.** In $\dfrac{(x+2)(x-3)}{(x+2)(x+5)}$, you can cancel $(x+2)$. But you cannot cancel individual terms inside parentheses.

Worksheet

1. Multiply and simplify. State all NPVs.

   (a) $\dfrac{x}{3} \times \dfrac{9}{x^2}$

   (b) $\dfrac{5x}{x-2} \times \dfrac{x-2}{10}$

   (c) $\dfrac{x+1}{x-4} \times \dfrac{x^2-16}{x^2-1}$

(d) $\dfrac{x^2-9}{x+1} \times \dfrac{x^2+x}{x-3}$

2. Divide and simplify. State all NPVs.

(a) $\dfrac{x^2}{4} \div \dfrac{x}{8}$

(b) $\dfrac{3x}{x+5} \div \dfrac{x}{x+5}$

(c) $\dfrac{x^2-4}{x+3} \div \dfrac{x-2}{x+3}$

(d) $\dfrac{x^2+3x}{x-1} \div \dfrac{x+3}{x^2-1}$

3. Multiply and simplify. State all NPVs.

(a) $\dfrac{x^2-5x+6}{x^2-4} \times \dfrac{x+2}{x-3}$

(b) $\dfrac{2x^2-8}{x+1} \times \dfrac{x^2-1}{x^2-4}$

4. Divide and simplify. State all NPVs.

   (a) $\dfrac{x^2-x-6}{x^2-9} \div \dfrac{x+2}{x+3}$

   (b) $\dfrac{3x^2-12}{x^2+x-2} \div \dfrac{x+2}{x-1}$

5. Simplify $\dfrac{x^2-1}{x^2-x-6} \times \dfrac{x^2-9}{x^2+4x+3}$. State all NPVs.

6. Simplify $\dfrac{x^2+5x+4}{x^2-16} \div \dfrac{x^2+x}{x-4}$. State all NPVs.

7. A student simplified $\dfrac{x^2-4}{x+2} \div \dfrac{x-2}{x+2}$ and got $(x-2)$ with $x \neq -2$. Identify all errors and state the correct answer.

8. **Challenge:** Simplify $\dfrac{2x^2+x-3}{x^2-1} \times \dfrac{x^2+2x+1}{2x^2+3x}$. State all NPVs.
   Hint: $2x^2+x-3$ *factors as* $(2x+3)(x-1)$ *and* $2x^2+3x = x(2x+3)$.

Answer Key

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1.

(a) $\dfrac{x}{3} \times \dfrac{9}{x^2}$

Step 1: No factoring needed. NPVs: denominator has x^2, so $x \neq 0$.

Step 2: Multiply and cancel.

$$\frac{x \times 9}{3 \times x^2} = \frac{9x}{3x^2} = \frac{3}{x}$$

$$\boxed{\frac{3}{x}, \quad x \neq 0}$$

(b) $\dfrac{5x}{x-2} \times \dfrac{x-2}{10}$

Step 1: NPVs: denominator has $(x-2)$, so $x \neq 2$.

Step 2: Cancel $(x-2)$ and reduce $5/10$.

$$\frac{5x \cdot (x-2)}{(x-2) \cdot 10} = \frac{5x}{10} = \frac{x}{2}$$

$$\boxed{\frac{x}{2}, \quad x \neq 2}$$

(c) $\dfrac{x+1}{x-4} \times \dfrac{x^2-16}{x^2-1}$

Step 1: Factor.

$$x^2 - 16 = (x-4)(x+4) \qquad x^2 - 1 = (x-1)(x+1)$$

Step 2: NPVs: denominators are $(x-4)$ and $(x^2-1) = (x-1)(x+1)$.

$$x \neq 4, \quad x \neq 1, \quad x \neq -1$$

Step 3: Rewrite and cancel $(x+1)$ and $(x-4)$.

$$\frac{(x+1)}{(x-4)} \times \frac{(x-4)(x+4)}{(x-1)(x+1)} = \frac{x+4}{x-1}$$

$$\boxed{\frac{x+4}{x-1}, \quad x \neq 4,\ x \neq 1,\ x \neq -1}$$

(d) $\dfrac{x^2-9}{x+1} \times \dfrac{x^2+x}{x-3}$

Step 1: Factor.

$$x^2 - 9 = (x-3)(x+3) \qquad x^2 + x = x(x+1)$$

Step 2: NPVs: denominators are $(x+1)$ and $(x-3)$.

$$x \neq -1, \quad x \neq 3$$

Step 3: Rewrite and cancel $(x+1)$ and $(x-3)$.

$$\frac{(x-3)(x+3)}{(x+1)} \times \frac{x(x+1)}{(x-3)} = x(x+3)$$

$$\boxed{x(x+3), \quad x \neq -1,\ x \neq 3}$$

2.

(a) $\dfrac{x^2}{4} \div \dfrac{x}{8}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2}{4} \times \frac{8}{x}$$

Step 2: NPVs: the original second denominator was 8 (no restriction), and the flipped denominator is x, so $x \neq 0$.

Step 3: Simplify.

$$\frac{x^2 \times 8}{4 \times x} = \frac{8x^2}{4x} = 2x$$

$$\boxed{2x, \quad x \neq 0}$$

(b) $\dfrac{3x}{x+5} \div \dfrac{x}{x+5}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{3x}{x+5} \times \frac{x+5}{x}$$

Step 2: NPVs: denominators are $(x+5)$ and x (after flipping).

$$x \neq -5, \quad x \neq 0$$

Step 3: Cancel $(x+5)$ and x.

$$\frac{3x}{(x+5)} \times \frac{(x+5)}{x} = 3$$

$$\boxed{3, \quad x \neq -5,\ x \neq 0}$$

(c) $\dfrac{x^2-4}{x+3} \div \dfrac{x-2}{x+3}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2-4}{x+3} \times \frac{x+3}{x-2}$$

Step 2: Factor. $x^2-4=(x-2)(x+2)$.

Step 3: NPVs: denominators are $(x+3)$ and $(x-2)$ (after flipping).

$$x \neq -3, \quad x \neq 2$$

Step 4: Cancel $(x+3)$ and $(x-2)$.

$$\frac{(x-2)(x+2)}{(x+3)} \times \frac{(x+3)}{(x-2)} = \frac{(x-2)(x+2)}{(x-2)} = x+2$$

$$\boxed{x+2, \quad x \neq -3, \ x \neq 2}$$

(d) $\dfrac{x^2+3x}{x-1} \div \dfrac{x+3}{x^2-1}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2+3x}{x-1} \times \frac{x^2-1}{x+3}$$

Step 2: Factor.

$$x^2+3x = x(x+3) \qquad x^2-1=(x-1)(x+1)$$

Step 3: NPVs: denominators are $(x-1)$, $(x+3)$ (after flipping), and $(x^2-1)=(x-1)(x+1)$ (after flipping).

$$x \neq 1, \quad x \neq -3, \quad x \neq -1$$

Step 4: Rewrite and cancel $(x+3)$ and $(x-1)$.

$$\frac{x(x+3)}{(x-1)} \times \frac{(x-1)(x+1)}{(x+3)} = x(x+1)$$

$$\boxed{x(x+1), \quad x \neq 1, \ x \neq -1, \ x \neq -3}$$

3.

(a) $\dfrac{x^2-5x+6}{x^2-4} \times \dfrac{x+2}{x-3}$

Step 1: Factor.

$$x^2-5x+6=(x-2)(x-3) \qquad x^2-4=(x-2)(x+2)$$

Step 2: NPVs: denominators are $(x^2-4)=(x-2)(x+2)$ and $(x-3)$.

$$x \neq 2, \quad x \neq -2, \quad x \neq 3$$

Step 3: Rewrite and cancel $(x-2)$, $(x-3)$, and $(x+2)$.

$$\frac{(x-2)(x-3)}{(x-2)(x+2)} \times \frac{(x+2)}{(x-3)} = 1$$

$$\boxed{1, \quad x \neq 2,\ x \neq -2,\ x \neq 3}$$

(b) $\dfrac{2x^2-8}{x+1} \times \dfrac{x^2-1}{x^2-4}$

Step 1: Factor.

$$2x^2-8 = 2(x^2-4) = 2(x-2)(x+2)$$

$$x^2-1=(x-1)(x+1) \qquad x^2-4=(x-2)(x+2)$$

Step 2: NPVs: denominators are $(x+1)$ and $(x^2-4)=(x-2)(x+2)$.

$$x \neq -1, \quad x \neq 2, \quad x \neq -2$$

Step 3: Rewrite and cancel $(x+1)$ and $(x^2-4)=(x-2)(x+2)$.

$$\frac{2(x-2)(x+2)}{(x+1)} \times \frac{(x-1)(x+1)}{(x-2)(x+2)} = 2(x-1)$$

$$\boxed{2(x-1), \quad x \neq -1,\ x \neq 2,\ x \neq -2}$$

4.

(a) $\dfrac{x^2-x-6}{x^2-9} \div \dfrac{x+2}{x+3}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2-x-6}{x^2-9} \times \frac{x+3}{x+2}$$

Step 2: Factor.

$$x^2-x-6=(x-3)(x+2) \qquad x^2-9=(x-3)(x+3)$$

Step 3: NPVs: all denominators are $(x^2-9)=(x-3)(x+3)$ and $(x+2)$ (after flipping).

$$x \neq 3, \quad x \neq -3, \quad x \neq -2$$

Step 4: Cancel $(x-3)$, $(x+2)$, and $(x+3)$.

$$\frac{(x-3)(x+2)}{(x-3)(x+3)} \times \frac{(x+3)}{(x+2)} = 1$$

$$\boxed{1, \quad x \neq 3,\ x \neq -3,\ x \neq -2}$$

(b) $\dfrac{3x^2-12}{x^2+x-2} \div \dfrac{x+2}{x-1}$

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{3x^2-12}{x^2+x-2} \times \frac{x-1}{x+2}$$

Step 2: Factor.

$$3x^2-12 = 3(x^2-4) = 3(x-2)(x+2)$$
$$x^2+x-2 = (x+2)(x-1)$$

Step 3: NPVs: denominators are $(x^2+x-2) = (x+2)(x-1)$ and $(x+2)$ (after flipping).

$$x \neq -2, \quad x \neq 1$$

Step 4: Cancel $(x+2)$ and $(x-1)$.

$$\frac{3(x-2)(x+2)}{(x+2)(x-1)} \times \frac{(x-1)}{(x+2)}$$

After cancelling $(x-1)$ and one $(x+2)$, there is still one $(x+2)$ in the denominator from the second fraction's original position. Let us redo carefully.

$$\frac{3(x-2)(x+2)}{(x+2)(x-1)} \times \frac{(x-1)}{(x+2)} = \frac{3(x-2)(x-1)}{(x-1)(x+2)} = \frac{3(x-2)}{(x-1)} \cdot \frac{(x-1)}{(x+2)} = \frac{3(x-2)}{x+2}$$

$$\boxed{\frac{3(x-2)}{x+2}, \quad x \neq -2,\ x \neq 1}$$

5. Simplify $\dfrac{x^2-1}{x^2-x-6} \times \dfrac{x^2-9}{x^2+4x+3}$.

Step 1: Factor everything.

$$x^2-1 = (x-1)(x+1)$$
$$x^2-x-6 = (x-3)(x+2)$$
$$x^2-9 = (x-3)(x+3)$$
$$x^2+4x+3 = (x+1)(x+3)$$

Step 2: NPVs from all denominators: $(x-3)(x+2)$ and $(x+1)(x+3)$.

$$x \neq 3, \quad x \neq -2, \quad x \neq -1, \quad x \neq -3$$

Step 3: Write all factors together and cancel.

$$\frac{(x-1)(x+1)(x-3)(x+3)}{(x-3)(x+2)(x+1)(x+3)} = \frac{x-1}{x+2}$$

$$\boxed{\frac{x-1}{x+2}, \quad x \neq 3,\ x \neq -2,\ x \neq -1,\ x \neq -3}$$

6. Simplify $\dfrac{x^2+5x+4}{x^2-16} \div \dfrac{x^2+x}{x-4}$.

Step 1: Rewrite as multiplication by the reciprocal.

$$\frac{x^2+5x+4}{x^2-16} \times \frac{x-4}{x^2+x}$$

Step 2: Factor.

$$x^2+5x+4 = (x+1)(x+4)$$
$$x^2-16 = (x-4)(x+4)$$
$$x^2+x = x(x+1)$$

Step 3: NPVs: denominators are $(x-4)(x+4)$ and $x(x+1)$ (after flipping). Also original second denominator was $(x-4)$, already captured.

$$x \neq 4, \quad x \neq -4, \quad x \neq 0, \quad x \neq -1$$

Step 4: Cancel $(x+4)$, $(x-4)$, and $(x+1)$.

$$\frac{(x+1)(x+4)}{(x-4)(x+4)} \times \frac{(x-4)}{x(x+1)} = \frac{1}{x}$$

$$\boxed{\frac{1}{x}, \quad x \neq 4,\ x \neq -4,\ x \neq 0,\ x \neq -1}$$

7. A student simplified $\dfrac{x^2-4}{x+2} \div \dfrac{x-2}{x+2}$ and got $(x-2)$ with $x \neq -2$.

Step 1: Rewrite correctly as multiplication by the reciprocal.

$$\frac{x^2-4}{x+2} \times \frac{x+2}{x-2}$$

Step 2: Factor. $x^2-4 = (x-2)(x+2)$.

Step 3: NPVs. Denominators at any point are $(x+2)$ and $(x-2)$ (after flipping).

$$x \neq -2 \quad \text{and} \quad x \neq 2$$

Step 4: Cancel $(x+2)$ and $(x-2)$.

$$\frac{(x-2)(x+2)}{(x+2)} \times \frac{(x+2)}{(x-2)} = 1$$

Step 5: State the student's errors. The student got $(x-2)$ instead of 1 — they likely forgot to cancel $(x-2)$ after flipping. Also, they missed the restriction $x \neq 2$.

$$\boxed{1, \quad x \neq -2 \text{ and } x \neq 2}$$

8. Simplify $\dfrac{2x^2+x-3}{x^2-1} \times \dfrac{x^2+2x+1}{2x^2+3x}$.

Step 1: Factor everything using the hint.

$$2x^2+x-3 = (2x+3)(x-1)$$

$$x^2-1 = (x-1)(x+1)$$

$$x^2+2x+1 = (x+1)^2$$

$$2x^2+3x = x(2x+3)$$

Step 2: NPVs from denominators $(x-1)(x+1)$ and $x(2x+3)$.

$$x \neq 1, \quad x \neq -1, \quad x \neq 0, \quad x \neq -\frac{3}{2}$$

Step 3: Write all factors and cancel $(2x+3)$, $(x-1)$, and one $(x+1)$.

$$\frac{(2x+3)(x-1)}{(x-1)(x+1)} \times \frac{(x+1)(x+1)}{x(2x+3)}$$

Wait — let us count carefully. Numerators have $(2x+3)$, $(x-1)$, $(x+1)^2$. Denominators have $(x-1)$, $(x+1)$, x, $(2x+3)$.

Cancel $(2x+3)$, $(x-1)$, and one $(x+1)$:

$$\frac{(x+1)}{x}$$

$$\boxed{\frac{x+1}{x}, \quad x \neq 1,\ x \neq -1,\ x \neq 0,\ x \neq -\frac{3}{2}}$$

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