Unit 4 Test
Systems of Equations & Inequalities
ApexMath

Name: ________________________________ **Date:** ________________

Instructions: Show all work clearly and in order. Write your final answer as an ordered pair (x, y) unless the question asks for something else. Part A requires answers only. Part B requires full step-by-step work.

Part A — Short Answer (2 points each, 20 points total)

Write only the answer. No work is required for Part A.

1. Solve using **substitution**.

$$\begin{cases} y = 2x - 1 \\ 3x + y = 9 \end{cases}$$

Answer: ____________________

2. Solve using **elimination**.

$$\begin{cases} 4x + y = 11 \\ x - y = 4 \end{cases}$$

Answer: ____________________

3. Solve using **substitution**. State the number of solutions.

$$\begin{cases} y = 3x + 2 \\ 6x - 2y = 5 \end{cases}$$

Answer: ____________________

4. Solve using **elimination**. State the number of solutions.

$$\begin{cases} 2x + 4y = 10 \\ x + 2y = 5 \end{cases}$$

Answer: ____________________

5. Solve the **linear-quadratic** system.

$$\begin{cases} y = x^2 - 2 \\ y = x + 4 \end{cases}$$

Answer: ______________________________

6. Solve the **linear-quadratic** system. State the number of solutions.

$$\begin{cases} y = x^2 + 5 \\ y = x + 3 \end{cases}$$

Answer: ____________________

7. Solve the **quadratic-quadratic** system.

$$\begin{cases} y = x^2 \\ y = -x^2 + 8 \end{cases}$$

Answer: ______________________________

8. Solve the **quadratic-quadratic** system.

$$\begin{cases} y = x^2 + 3x \\ y = x^2 - x + 8 \end{cases}$$

Answer: ____________________

9. Consider the system of inequalities.

$$\begin{cases} y > 2x - 1 \\ y \leq -x + 4 \end{cases}$$

Is the point $(1, 3)$ a solution to this system? Circle one: **Yes** **No**

10. Consider the system of inequalities.

$$\begin{cases} y \geq x^2 - 1 \\ y < 3 \end{cases}$$

Is the point $(3, 1)$ a solution to this system? Circle one: **Yes** **No**

Part B — Full Solutions

(16 points each, 80 points total)

Show every step clearly. Unsupported answers will not receive full credit.

B1. Solve the system using **substitution**. Verify your answer in both equations.

$$\begin{cases} 3x + 2y = 16 \\ y = x - 2 \end{cases}$$

B2. Solve the system using **elimination**. Show the multiplication step clearly.

$$\begin{cases} 3x + 4y = 10 \\ 5x - 2y = 8 \end{cases}$$

B3. Solve the linear-quadratic system. Show the discriminant check and verify both solutions.

$$\begin{cases} y = x^2 - 4x + 3 \\ y = x - 1 \end{cases}$$

B4. Solve the quadratic-quadratic system. Show the subtraction step clearly.

$$\begin{cases} y = x^2 - 2 \\ y = -x^2 + 6 \end{cases}$$

B5. For the system of inequalities below, do the following:

(a) Find the boundary intersection points.

(b) State whether each boundary is solid or dashed, and which side to shade for each inequality.

(c) Describe the solution region in words.

(d) Verify your solution region with a test point.

$$\begin{cases} y \geq x^2 - 4 \\ y < 2x + 4 \end{cases}$$

Answer Key

Part A

A1.

$$y = 2x - 1 \text{ already solved. Sub into } 3x + y = 9: \quad 3x + (2x - 1) = 9 \implies 5x = 10 \implies x = 2, \quad y = 3$$

$$\boxed{(2,\ 3)}$$

A2.

$$y\text{-coefficients are } +1 \text{ and } -1. \text{ Add directly:} \quad (4x + y) + (x - y) = 11 + 4 \implies 5x = 15 \implies x = 3$$

$$x - y = 4 \implies 3 - y = 4 \implies y = -1$$

$$\boxed{(3,\ -1)}$$

A3.

Substitute $y = 3x + 2$ into $6x - 2y = 5$:

$$6x - 2(3x + 2) = 5 \implies 6x - 6x - 4 = 5 \implies -4 = 5$$

Never true. The lines are parallel.

$$\boxed{\text{No solution}}$$

A4.

Multiply the second equation by -2: $-2x - 4y = -10$. Add to the first equation:

$$(2x + 4y) + (-2x - 4y) = 10 + (-10) \implies 0 = 0$$

Always true. Both equations describe the same line.

$$\boxed{\text{Infinitely many solutions}}$$

A5.

Set equal: $x^2 - 2 = x + 4 \implies x^2 - x - 6 = 0$.

$$\Delta = 1 + 24 = 25 > 0 \qquad (x - 3)(x + 2) = 0 \implies x = 3 \text{ or } x = -2$$

$$x = 3: \ y = 3 + 4 = 7 \qquad x = -2: \ y = -2 + 4 = 2$$

$$\boxed{(3,\ 7) \text{ and } (-2,\ 2)}$$

A6.

Set equal: $x^2 + 5 = x + 3 \implies x^2 - x + 2 = 0.$

$$\Delta = (-1)^2 - 4(1)(2) = 1 - 8 = -7 < 0$$

No real solutions.

$$\boxed{\text{No solution}}$$

A7.

Subtract Equation 2 from Equation 1:

$$0 = x^2 - (-x^2 + 8) = 2x^2 - 8 \implies x^2 = 4 \implies x = \pm 2$$

$$x = 2: \; y = (2)^2 = 4 \qquad x = -2: \; y = (-2)^2 = 4$$

$$\boxed{(2,\ 4) \text{ and } (-2,\ 4)}$$

A8.

Subtract Equation 2 from Equation 1:

$$0 = (x^2 + 3x) - (x^2 - x + 8) = 4x - 8 \implies x = 2$$

$$y = (2)^2 + 3(2) = 4 + 6 = 10$$

$$\boxed{(2,\ 10)}$$

A9.

Test $(1, 3)$:

$$y > 2x - 1: \quad 3 > 2(1) - 1 = 1 \quad \checkmark$$

$$y \leq -x + 4: \quad 3 \leq -1 + 4 = 3 \quad \checkmark$$

$$\boxed{\textbf{Yes}}$$

A10.

Test $(3, 1)$:

$$y \geq x^2 - 1: \quad 1 \geq (3)^2 - 1 = 8 \quad \text{False}$$

The first inequality fails immediately.

$$\boxed{\textbf{No}}$$

Part B

B1. $\begin{cases} 3x + 2y = 16 \\ y = x - 2 \end{cases}$

Step 1: The second equation is already solved for y: $y = x - 2$.

Step 2: Substitute $y = x - 2$ into the first equation.

$$3x + 2(x - 2) = 16$$

Step 3: Expand and solve.

$$3x + 2x - 4 = 16$$
$$5x - 4 = 16$$
$$5x = 20$$
$$x = 4$$

Step 4: Substitute $x = 4$ into $y = x - 2$.

$$y = 4 - 2 = 2$$

Step 5: Verify in both equations.

$$\text{Eq. 1: } 3(4) + 2(2) = 12 + 4 = 16 \quad \checkmark$$
$$\text{Eq. 2: } y = 4 - 2 = 2 \quad \checkmark$$

$$\boxed{(4,\ 2)}$$

B2. $\begin{cases} 3x + 4y = 10 \\ 5x - 2y = 8 \end{cases}$

Step 1: Eliminate y. The y-coefficients are $+4$ and -2. Multiply the second equation by 2 to make them $+4$ and -4.

$$2(5x - 2y) = 2(8) \implies 10x - 4y = 16$$

Step 2: Add to the first equation.

$$(3x + 4y) + (10x - 4y) = 10 + 16$$
$$13x = 26$$
$$x = 2$$

Step 3: Substitute $x = 2$ into the first equation.

$$3(2) + 4y = 10 \implies 6 + 4y = 10 \implies 4y = 4 \implies y = 1$$

Step 4: Verify.

$$\text{Eq. 1: } 3(2) + 4(1) = 6 + 4 = 10 \quad \checkmark$$
$$\text{Eq. 2: } 5(2) - 2(1) = 10 - 2 = 8 \quad \checkmark$$

$$\boxed{(2,\ 1)}$$

B3. $\begin{cases} y = x^2 - 4x + 3 \\ y = x - 1 \end{cases}$

Step 1: Set the right-hand sides equal.

$$x^2 - 4x + 3 = x - 1$$

Step 2: Rearrange into standard form.

$$x^2 - 4x + 3 - x + 1 = 0$$
$$x^2 - 5x + 4 = 0$$

Step 3: Discriminant check.

$$\Delta = (-5)^2 - 4(1)(4) = 25 - 16 = 9 > 0 \quad \text{(two solutions)}$$

Step 4: Factor.

$$(x - 4)(x - 1) = 0 \implies x = 4 \quad \text{or} \quad x = 1$$

Step 5: Find y using $y = x - 1$.

$$x = 4: \quad y = 4 - 1 = 3$$
$$x = 1: \quad y = 1 - 1 = 0$$

Step 6: Verify both solutions.

$$(4, 3): \quad y = (4)^2 - 4(4) + 3 = 16 - 16 + 3 = 3 \quad \checkmark \quad y = 4 - 1 = 3 \quad \checkmark$$
$$(1, 0): \quad y = (1)^2 - 4(1) + 3 = 1 - 4 + 3 = 0 \quad \checkmark \quad y = 1 - 1 = 0 \quad \checkmark$$

$$\boxed{(4,\ 3) \text{ and } (1,\ 0)}$$

B4. $\begin{cases} y = x^2 - 2 \\ y = -x^2 + 6 \end{cases}$

Step 1: Subtract Equation 2 from Equation 1 to eliminate y.

$$0 = (x^2 - 2) - (-x^2 + 6)$$

$$0 = x^2 - 2 + x^2 - 6$$

$$0 = 2x^2 - 8$$

Step 2: Solve for x.

$$2x^2 = 8 \implies x^2 = 4 \implies x = 2 \quad \text{or} \quad x = -2$$

Step 3: Substitute into $y = x^2 - 2$.

$$x = 2: \quad y = (2)^2 - 2 = 4 - 2 = 2$$

$$x = -2: \quad y = (-2)^2 - 2 = 4 - 2 = 2$$

Step 4: Verify both solutions.

$$(2, 2): \quad y = 4 - 2 = 2 \quad \checkmark \quad y = -4 + 6 = 2 \quad \checkmark$$

$$(-2, 2): \quad y = 4 - 2 = 2 \quad \checkmark \quad y = -4 + 6 = 2 \quad \checkmark$$

$$\boxed{(2,\ 2) \text{ and } (-2,\ 2)}$$

B5. $\begin{cases} y \geq x^2 - 4 \\ y < 2x + 4 \end{cases}$

(a) Find the boundary intersection points.

Set $x^2 - 4 = 2x + 4$:

$$x^2 - 2x - 8 = 0 \implies (x - 4)(x + 2) = 0$$

$$x = 4, \quad y = 12 \qquad \text{and} \qquad x = -2, \quad y = 0$$

Intersection points: $(-2,\ 0)$ and $(4,\ 12)$.

(b) Boundary types and shading.

- $y = x^2 - 4$: **solid** curve ($\geq$ includes the boundary). Shade *above* the parabola — inside the cup.
- $y = 2x + 4$: **dashed** line ($<$ excludes the boundary). Shade *below* the line.

(c) Solution region.

The solution is the bounded region *above or on* the solid parabola and *below* the dashed line, between the intersection points $(-2, 0)$ and $(4, 12)$.

(d) Verify with test point $(0, 0)$**:**

$$y \geq x^2 - 4: \quad 0 \geq 0 - 4 = -4 \quad \checkmark$$

$$y < 2x + 4: \quad 0 < 2(0) + 4 = 4 \quad \checkmark$$

$(0, 0)$ satisfies both inequalities and lies in the described region.

$$\boxed{\text{Bounded region above/on } y = x^2 - 4 \text{ and below } y = 2x + 4, \text{ between } (-2, 0) \text{ and } (4, 12)}$$

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