Unit 8 Review: Trigonometry
Sine and Cosine Laws
ApexMath

This review covers all seven lessons in Unit 8. For each problem, draw a labelled diagram, identify the configuration, state which law you are using, and show all steps clearly.

Formula Sheet

Special Triangles: $1 : 1 : \sqrt{2}$ (45-45-90) $1 : \sqrt{3} : 2$ (30-60-90)

Sine Law: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Ambiguous Case: Compute $h = b \sin A$; compare a to h and b

Cosine Law (side): $a^2 = b^2 + c^2 - 2bc \cos A$

Cosine Law (angle): $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area: $\text{Area} = \dfrac{1}{2}\, b\, c\, \sin A$

Given	**Use**	**Note**
AAS or ASA	Sine Law	Find third angle first
SSA	Sine Law	Check ambiguous case
SAS	Cosine Law $\rightarrow$ Sine Law	Negative cos if $A > 90$
SSS	Cosine Law $\rightarrow$ Sine Law	Find largest angle first

Section 1: Special Triangles and Exact Values

1. A 30-60-90 triangle has a hypotenuse of length 16. Find the exact lengths of both legs.

2. A 45-45-90 triangle has a hypotenuse of length $10\sqrt{2}$. Find the exact length of each leg.

3. Without a calculator, find the exact value of:

$$\sin 60 \times \tan 30 - \cos^2 45$$

Section 2: The Sine Law

4. In triangle ABC, $A = 48$, $B = 65$, and $a = 22$. Find sides b and c.

5. In triangle ABC, $a = 20$, $b = 15$, and $A = 72$. Determine how many triangles are possible, then solve.

6. **(Real Life)** A surveyor uses two angles to locate a landmark. From point A, the angle to the landmark is 55 from the baseline AB. From point B, the angle is 72. The baseline $AB = 8$ km. Find the distance from each point to the landmark.

7. **(Ambiguous Case)** In triangle ABC, $a = 11$, $b = 16$, and $A = 40$. Determine how many triangles exist and solve all of them completely.

Section 3: The Cosine Law

8. In triangle ABC, $b = 9$, $c = 14$, and $A = 58$. Find side a, then find angles B and C.

9. In triangle ABC, $a = 7$, $b = 10$, $c = 13$. Find all three angles.

10. In triangle ABC, $b = 11$, $c = 15$, and $A = 115$. Find side a, then find angles B and C.

Section 4: Applications

11. (Real Life — Bearing) Two ships leave the same harbour. Ship A travels 30 km on a bearing of N25°E and Ship B travels 45 km on a bearing of N70°E. How far apart are the two ships?

12. (Real Life — Area) A triangular plot of land has two sides of 16 m and 22 m meeting at an angle of 70. Find the area of the plot.

13. (Real Life — Indirect Measurement) Two observers stand 300 m apart on flat ground, both watching a tower directly ahead. The angle from observer A to the top of the tower is 68 from the baseline. The angle from observer B is 55. Find the distance from each observer to the base of the tower.

14. (Real Life — SSS) Three towns form a triangle. The distances between them are 50 km, 70 km, and 90 km. Find all three angles of the triangle.

15. (Challenge — Ambiguous Case) In a navigation problem, $a = 12$ km, $b = 18$ km, and $A = 38$. Solve all possible triangles completely. For each valid triangle, state all three angles and all three sides.

16. (Challenge — Parallelogram) A parallelogram has sides of 10 cm and 15 cm. The angle between the sides is 72.

- Find the lengths of both diagonals.
- Find the area of the parallelogram.

Answer Key

1. 30-60-90, hypotenuse $= 16$.

Step 1: The hypotenuse corresponds to $2k$. Solve for k.

$$2k = 16 \qquad \Longrightarrow \qquad k = 8$$

Step 2: Apply the ratios.

$$\text{short leg} = k = 8 \qquad \text{long leg} = k\sqrt{3} = 8\sqrt{3}$$

$$\boxed{\text{short leg} = 8, \quad \text{long leg} = 8\sqrt{3} \approx 13.86}$$

2. 45-45-90, hypotenuse $= 10\sqrt{2}$.

Step 1: Solve for k.

$$k\sqrt{2} = 10\sqrt{2} \qquad \Longrightarrow \qquad k = 10$$

$$\boxed{\text{each leg} = 10}$$

3. Evaluate $\sin 60 \times \tan 30 - \cos^2 45$ exactly.

Step 1: Substitute exact values.

$$\sin 60 = \frac{\sqrt{3}}{2}, \qquad \tan 30 = \frac{\sqrt{3}}{3}, \qquad \cos 45 = \frac{\sqrt{2}}{2}$$

Step 2: Evaluate each part.

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} = \frac{3}{6} = \frac{1}{2}$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

Step 3: Subtract.

$$\frac{1}{2} - \frac{1}{2}$$

$$\boxed{0}$$

4. $A = 48$, $B = 65$, $a = 22$. AAS — Sine Law.

Step 1: Find C.

$$C = 180 - 48 - 65 = 67$$

Step 2: Find b.

$$b = \frac{22 \times \sin 65}{\sin 48} = \frac{22 \times 0.9063}{0.7431} \approx 26.83$$

Step 3: Find c.

$$c = \frac{22 \times \sin 67}{\sin 48} = \frac{22 \times 0.9205}{0.7431} \approx 27.25$$

$$\boxed{C = 67, \quad b \approx 26.83, \quad c \approx 27.25}$$

5. $a = 20$, $b = 15$, $A = 72$. SSA.

Step 1: Compute h.

$$h = 15 \times \sin 72 = 15 \times 0.9511 = 14.27$$

Since $a = 20 \geq b = 15$, exactly **one triangle** exists.
Step 2: Find B.

$$\sin B = \frac{15 \times \sin 72}{20} = \frac{15 \times 0.9511}{20} = 0.7133$$

$$B = \sin^{-1}(0.7133) \approx 45.50$$

Step 3: Find C and c.

$$C = 180 - 72 - 45.50 = 62.50$$

$$c = \frac{20 \times \sin 62.50}{\sin 72} = \frac{20 \times 0.8870}{0.9511} \approx 18.65$$

$$\boxed{B \approx 45.50, \quad C \approx 62.50, \quad c \approx 18.65}$$

6. $A = 55$, $B = 72$, $AB = 8$ km. AAS — Sine Law.

Step 1: Find C.

$$C = 180 - 55 - 72 = 53$$

Step 2: Find AC (distance from A to landmark, opposite B).

$$AC = \frac{8 \times \sin 72}{\sin 53} = \frac{8 \times 0.9511}{0.7986} \approx 9.53 \text{ km}$$

Step 3: Find BC (distance from B to landmark, opposite A).

$$BC = \frac{8 \times \sin 55}{\sin 53} = \frac{8 \times 0.8192}{0.7986} \approx 8.21 \text{ km}$$

$$\boxed{AC \approx 9.53 \text{ km}, \quad BC \approx 8.21 \text{ km}}$$

7. $a = 11$, $b = 16$, $A = 40$. SSA — Ambiguous Case.

Step 1: Compute h.

$$h = 16 \times \sin 40 = 16 \times 0.6428 = 10.28$$

Since $h = 10.28 \leq a = 11 < b = 16$, **two triangles** exist.
Step 2: Find $\sin B$.

$$\sin B = \frac{16 \times \sin 40}{11} = \frac{16 \times 0.6428}{11} = 0.9350$$

Triangle 1:

$$B_1 = \sin^{-1}(0.9350) \approx 69.22, \quad C_1 = 180 - 40 - 69.22 = 70.78$$

$$c_1 = \frac{11 \times \sin 70.78}{\sin 40} = \frac{11 \times 0.9434}{0.6428} \approx 16.16$$

Triangle 2:

$$B_2 = 180 - 69.22 = 110.78, \quad C_2 = 180 - 40 - 110.78 = 29.22$$

$$c_2 = \frac{11 \times \sin 29.22}{\sin 40} = \frac{11 \times 0.4879}{0.6428} \approx 8.35$$

$$\boxed{\begin{array}{ll} \textbf{Triangle 1:} & B_1 \approx 69.22,\ C_1 \approx 70.78,\ c_1 \approx 16.16 \\ \textbf{Triangle 2:} & B_2 \approx 110.78,\ C_2 \approx 29.22,\ c_2 \approx 8.35 \end{array}}$$

8. $b = 9$, $c = 14$, $A = 58$. SAS.

Step 1: Find a.

$$a^2 = 9^2 + 14^2 - 2(9)(14)\cos 58 = 81 + 196 - 252 \times 0.5299 = 277 - 133.53 = 143.47$$

$$a = \sqrt{143.47} \approx 11.98$$

Step 2: Find B.

$$\sin B = \frac{9 \times \sin 58}{11.98} = \frac{9 \times 0.8480}{11.98} = 0.6372$$

$$B = \sin^{-1}(0.6372) \approx 39.59$$

Step 3: Find C.

$$C = 180 - 58 - 39.59 = 82.41$$

$$\boxed{a \approx 11.98, \quad B \approx 39.59, \quad C \approx 82.41}$$

9. $a = 7$, $b = 10$, $c = 13$. SSS.

Step 1: Find C (largest side $c = 13$).

$$\cos C = \frac{7^2 + 10^2 - 13^2}{2(7)(10)} = \frac{49 + 100 - 169}{140} = \frac{-20}{140} = -0.1429$$

$$C = \cos^{-1}(-0.1429) \approx 98.21$$

Step 2: Find B.

$$\cos B = \frac{7^2 + 13^2 - 10^2}{2(7)(13)} = \frac{49 + 169 - 100}{182} = \frac{118}{182} = 0.6484$$

$$B = \cos^{-1}(0.6484) \approx 49.58$$

Step 3: Find A.

$$A = 180 - 98.21 - 49.58 = 32.21$$

$$\boxed{A \approx 32.21, \quad B \approx 49.58, \quad C \approx 98.21}$$

10. $b = 11$, $c = 15$, $A = 115$. SAS.

Step 1: Note $\cos 115 \approx -0.4226$. Find a.

$$a^2 = 11^2 + 15^2 - 2(11)(15)\cos 115 = 121 + 225 - 330 \times (-0.4226) = 346 + 139.46 = 485.46$$

$$a = \sqrt{485.46} \approx 22.03$$

Step 2: Find B (use smaller side to avoid ambiguity).

$$\sin B = \frac{11 \times \sin 115}{22.03} = \frac{11 \times 0.9063}{22.03} = 0.4527$$

$$B = \sin^{-1}(0.4527) \approx 26.90$$

Step 3: Find C.

$$C = 180 - 115 - 26.90 = 38.10$$

$$\boxed{a \approx 22.03, \quad B \approx 26.90, \quad C \approx 38.10}$$

11. Ships: 30 km on N25°E and 45 km on N70°E.

Step 1: Angle between paths $= 70 - 25 = 45$**. SAS.**

$$d^2 = 30^2 + 45^2 - 2(30)(45)\cos 45 = 900 + 2025 - 2700 \times 0.7071 = 2925 - 1909.14 = 1015.86$$

$$d = \sqrt{1015.86}$$

$$\boxed{d \approx 31.87 \text{ km}}$$

12. Two sides 16 m and 22 m, included angle 70.

$$\text{Area} = \frac{1}{2} \times 16 \times 22 \times \sin 70 = 176 \times 0.9397$$

$$\boxed{\text{Area} \approx 165.39 \text{ m}^2}$$

13. Two observers 300 m apart. Angles 68 and 55.

Step 1: Find the angle at the tower.

$$T = 180 - 68 - 55 = 57$$

Step 2: AAS. $AB = 300$ **m opposite** T**.**

$$AT = \frac{300 \times \sin 55}{\sin 57} = \frac{300 \times 0.8192}{0.8387} \approx 293.02 \text{ m}$$

$$BT = \frac{300 \times \sin 68}{\sin 57} = \frac{300 \times 0.9272}{0.8387} \approx 331.66 \text{ m}$$

$$\boxed{AT \approx 293.02 \text{ m}, \quad BT \approx 331.66 \text{ m}}$$

14. Sides 50 km, 70 km, 90 km. SSS.

Step 1: Find C **(largest side** $c = 90$**).**

$$\cos C = \frac{50^2 + 70^2 - 90^2}{2(50)(70)} = \frac{2500 + 4900 - 8100}{7000} = \frac{-700}{7000} = -0.1000$$

$$C = \cos^{-1}(-0.1000) \approx 95.74$$

Step 2: Find B.

$$\cos B = \frac{50^2 + 90^2 - 70^2}{2(50)(90)} = \frac{2500 + 8100 - 4900}{9000} = \frac{5700}{9000} = 0.6333$$

$$B = \cos^{-1}(0.6333) \approx 50.70$$

Step 3: Find A.

$$A = 180 - 95.74 - 50.70 = 33.56$$

$$\boxed{A \approx 33.56, \quad B \approx 50.70, \quad C \approx 95.74}$$

15. (Challenge) $a = 12$ km, $b = 18$ km, $A = 38$.

Step 1: Compute h.

$$h = 18 \times \sin 38 = 18 \times 0.6157 = 11.08$$

Since $h = 11.08 \leq a = 12 < b = 18$, **two triangles** exist.
Step 2: Find $\sin B$.

$$\sin B = \frac{18 \times \sin 38}{12} = \frac{18 \times 0.6157}{12} = 0.9236$$

Triangle 1:

$$B_1 = \sin^{-1}(0.9236) \approx 67.44, \quad C_1 = 180 - 38 - 67.44 = 74.56$$

$$c_1 = \frac{12 \times \sin 74.56}{\sin 38} = \frac{12 \times 0.9639}{0.6157} \approx 18.79 \text{ km}$$

Triangle 2:

$$B_2 = 180 - 67.44 = 112.56, \quad C_2 = 180 - 38 - 112.56 = 29.44$$

$$c_2 = \frac{12 \times \sin 29.44}{\sin 38} = \frac{12 \times 0.4915}{0.6157} \approx 9.58 \text{ km}$$

$$\boxed{\begin{array}{ll} \textbf{Triangle 1:} & B_1 \approx 67.44,\ C_1 \approx 74.56,\ c_1 \approx 18.79 \text{ km} \\ \textbf{Triangle 2:} & B_2 \approx 112.56,\ C_2 \approx 29.44,\ c_2 \approx 9.58 \text{ km} \end{array}}$$

16. (Challenge) Parallelogram: sides 10 cm and 15 cm, angle 72.

Step 1: Find diagonal d_1 (included angle 72).

$$d_1^2 = 10^2 + 15^2 - 2(10)(15)\cos 72 = 100 + 225 - 300 \times 0.3090 = 325 - 92.71 = 232.29$$

$$d_1 = \sqrt{232.29} \approx 15.24 \text{ cm}$$

Step 2: Find diagonal d_2 (included angle $180 - 72 = 108$).

$$d_2^2 = 10^2 + 15^2 - 2(10)(15)\cos 108 = 325 - 300 \times (-0.3090) = 325 + 92.71 = 417.71$$

$$d_2 = \sqrt{417.71} \approx 20.44 \text{ cm}$$

Step 3: Find the area.

$$\text{Area} = 10 \times 15 \times \sin 72 = 150 \times 0.9511$$

$$\boxed{d_1 \approx 15.24 \text{ cm}, \quad d_2 \approx 20.44 \text{ cm}, \quad \text{Area} \approx 142.66 \text{ cm}^2}$$

Thank you for learning with ApexMath!

If you found this lesson helpful, we'd love for you to share your feedback or leave a review.

End of Unit 8 Review • ApexMath
