Pre-Calculus 12 – Complete Study Notes
Lesson 3: Polynomials Using Synthetic Division

Notes: Synthetic Division, Remainder, and Factor Theorems

Synthetic Division

Synthetic division is a shortcut method for dividing a polynomial by a linear binomial of the form $(x - c)$. It is faster than long division and especially useful for checking potential factors.

Steps for Synthetic Division:

1. Write the coefficients of the polynomial in descending powers of x, including 0 placeholders for missing terms.
2. Write c (from $x - c$) on the left side.
3. Bring down the first coefficient.
4. Multiply c by the number just written below the line; write the result under the next coefficient.
5. Add the column, repeat multiplication and addition until the last column.
6. The last number is the remainder; the other numbers are the coefficients of the quotient polynomial.

Remainder Theorem

If a polynomial $f(x)$ is divided by $(x - c)$, the remainder is $f(c)$. If $f(c) = 0$, then $x - c$ is a factor of the polynomial.

Factor Theorem

The Factor Theorem is a special case of the Remainder Theorem:

- If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.
- If $f(c) \neq 0$, then $x - c$ is not a factor.

Walkthrough of Example Question

Example: Fully factor $P(x) = 2x^3 - 3x^2 - 11x + 6$ using synthetic division.

Step 1: Find a possible root using the Rational Root Theorem. Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$.

Test $f(1)$:

$$f(1) = 2(1)^3 - 3(1)^2 - 11(1) + 6 = 2 - 3 - 11 + 6 = -6 \quad \text{(not zero)}$$

Test $f(2)$:

$$f(2) = 2(8) - 3(4) - 11(2) + 6 = 16 - 12 - 22 + 6 = -12 \quad \text{(not zero)}$$

Test $f(-1)$:

$$f(-1) = 2(-1)^3 - 3(1) - 11(-1) + 6 = -2 - 3 + 11 + 6 = 12 \quad \text{(not zero)}$$

Test $f(3)$:

$$f(3) = 2(27) - 3(9) - 11(3) + 6 = 54 - 27 - 33 + 6 = 0$$

So $x - 3$ is a factor.

Step 2: Synthetic division to divide by $(x - 3)$:

$$\begin{array}{c|cccc} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

Quotient: $2x^2 + 3x - 2$ Remainder: 0 (confirms factor).

Step 3: Factor the quadratic $2x^2 + 3x - 2$:

$$2x^2 + 3x - 2 = (2x - 1)(x + 2)$$

Step 4: Final factorization:

$$P(x) = (x - 3)(2x - 1)(x + 2)$$

Worksheet: Practice Questions

1. Fully factor $x^3 - 6x^2 + 11x - 6$.
2. Use the Remainder Theorem to find the remainder when $2x^3 - 5x^2 + 4x - 7$ is divided by $(x - 2)$.
3. Determine if $(x + 4)$ is a factor of $3x^3 + 5x^2 - 2x - 8$.
4. Factor completely: $2x^3 + x^2 - 13x + 6$.

Answer Key

Q1: $x^3 - 6x^2 + 11x - 6$ Possible roots: $\pm 1, \pm 2, \pm 3, \pm 6$.
Test $f(1) = 1 - 6 + 11 - 6 = 0 \Rightarrow$ factor is $(x - 1)$.
Synthetic division:

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

Quotient: $x^2 - 5x + 6 = (x - 2)(x - 3)$
Final: $(x - 1)(x - 2)(x - 3)$
Q2: $2x^3 - 5x^2 + 4x - 7$ divided by $(x - 2)$
Remainder theorem:

$$f(2) = 2(8) - 5(4) + 4(2) - 7 = 16 - 20 + 8 - 7 = -3$$

Remainder $= -3$
Q3: Test $(x + 4)$ for $3x^3 + 5x^2 - 2x - 8$
$c = -4$, $f(-4) = 3(-64) + 5(16) - 2(-4) - 8 = -192 + 80 + 8 - 8 = -112 \neq 0$ Conclusion: Not a factor.
Q4: $2x^3 + x^2 - 13x + 6$ Possible roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$
Test $x = 2$: $f(2) = 16 + 4 - 26 + 6 = 0 \Rightarrow$ factor is $(x - 2)$
Synthetic division:

$$\begin{array}{c|cccc} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

Quotient: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$
Final factorization: $(x - 2)(2x - 1)(x + 3)$