

# Pre-Calculus 12 – Complete Study Notes

## Lesson 1: Arithmetic and Geometric Sequences and Series

### Notes: Arithmetic and Geometric Sequences and Series

A **sequence** is an ordered list of numbers. Each number in the sequence is called a **term**. A **series** is the sum of terms of a sequence.

#### 1. Arithmetic Sequences (A.S.)

- Difference between consecutive terms is constant:  $d = a_{n+1} - a_n$  - General term:  $a_n = a_1 + (n - 1)d$  - Sum of first  $n$  terms:  $S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n - 1)d]$

#### 2. Geometric Sequences (G.S.)

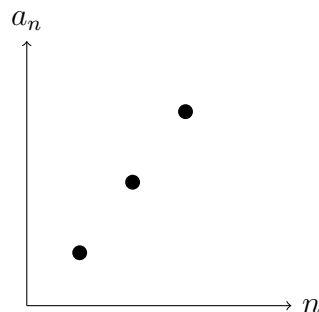
- Ratio between consecutive terms is constant:  $r = \frac{a_{n+1}}{a_n}$  - General term:  $a_n = a_1 \cdot r^{n-1}$  - Sum of first  $n$  terms:  $S_n = a_1 \frac{1-r^n}{1-r}$ ,  $r \neq 1$  - Infinite sum ( $|r| < 1$ ):  $S_\infty = \frac{a_1}{1-r}$

#### 3. Ways to Represent a Relation or Function

- **Table**

|       |   |   |    |
|-------|---|---|----|
| $n$   | 1 | 2 | 3  |
| $a_n$ | 3 | 7 | 11 |

- **Graph**



- **Equation**

$$a_n = a_1 + (n - 1)d$$

## Walkthrough of Example Questions

**Example 1:** Find the 10th term of the arithmetic sequence 2, 5, 8, ...

**Solution:**

$$\begin{aligned}a_1 &= 2, & d &= 5 - 2 = 3 \\a_{10} &= a_1 + (10 - 1)d = 2 + 9(3) = 2 + 27 = 29\end{aligned}$$

**Example 2:** Find the sum of the first 15 terms of the arithmetic sequence 4, 7, 10, ...

**Solution:**

$$\begin{aligned}a_1 &= 4, & d &= 7 - 4 = 3, & n &= 15 \\a_{15} &= a_1 + (15 - 1)d = 4 + 14(3) = 46 \\S_{15} &= \frac{15}{2}(a_1 + a_{15}) = \frac{15}{2}(4 + 46) = \frac{15}{2}(50) = 375\end{aligned}$$

**Example 3:** Find the sum of the first 8 terms of the geometric sequence 3, 6, 12, ...

**Solution:**

$$\begin{aligned}a_1 &= 3, & r &= \frac{6}{3} = 2, & n &= 8 \\S_8 &= a_1 \frac{1 - r^n}{1 - r} = 3 \frac{1 - 2^8}{1 - 2} = 3 \frac{1 - 256}{-1} = 3 \cdot 255 = 765\end{aligned}$$

**Example 4 (Bouncing Ball):** A ball is dropped from 120 cm and bounces to 75% of its previous height each time. Find the total distance traveled after 4 bounces.

**Solution:**

$$\text{Total distance} = h_1 + 2 \frac{a(1 - r^n)}{1 - r}, \quad h_1 = 120, \quad a = 120 \cdot 0.75 = 90, \quad r = 0.75, \quad n = 4$$

$$\begin{aligned}\text{Total distance} &= 120 + 2 \frac{90(1 - 0.75^4)}{1 - 0.75} = 120 + 2 \frac{90(1 - 0.31640625)}{0.25} \\&= 120 + 2 \frac{90 \cdot 0.68359375}{0.25} = 120 + 2 \cdot 246.09375 = 120 + 492.1875 \approx 612.19 \text{ cm}\end{aligned}$$

*Explanation:* First drop counts once, then each bounce up and down is doubled. The geometric series formula is used to sum the bounces.

## Worksheet: Practice Questions

1. Find the 12th term of the arithmetic sequence 5, 9, 13, ...
2. Find the sum of the first 20 terms of the arithmetic sequence 1, 4, 7, ...

3. Find the 6th term of the geometric sequence  $2, 4, 8, \dots$
4. Find the sum of the first 10 terms of the geometric sequence  $1, 3, 9, \dots$
5. Determine whether the sequence  $2, 4, 8, 16, \dots$  is arithmetic or geometric.
6. If a geometric sequence has first term 5 and ratio  $r = 3$ , find the 4th term.
7. A ball is dropped from 120 cm and bounces to 75% of its previous height each time. Find the total distance traveled after 4 bounces.

## Answer Key

**Q1:**  $a_{12} = 5 + (12 - 1) \cdot 4 = 5 + 44 = 49$

**Q2:**  $S_{20} = \frac{20}{2}[2 + (20 - 1)3] = 10(2 + 57) = 10(59) = 590$

**Q3:**  $a_6 = 2 \cdot 2^5 = 2 \cdot 32 = 64$

**Q4:**  $S_{10} = 1 \frac{1-3^{10}}{1-3} = \frac{1-59049}{-2} = 29524$

**Q5:** Geometric: ratio is constant  $r = 2$

**Q6:**  $a_4 = 5 \cdot 3^3 = 5 \cdot 27 = 135$

**Q7:** Total distance:  $h_1 = 120, a = 120 \cdot 0.75 = 90, r = 0.75, n = 4$   
 Total distance  $D = h_1 + 2 \frac{a(1-r^n)}{1-r} = 120 + 2 \frac{90(1-0.75^4)}{0.25} = 120 + 492.1875 \approx 612.19$  cm