

Pre-Calculus 12 – Complete Study Notes

Lesson 2: Transformations

Notes: Transformations of Functions

Transformations modify the appearance or position of a graph of a function. They can shift, stretch, compress, or reflect the graph.

1. Vertical Shifts

- $f(x) + k$ shifts the graph up if $k > 0$ and down if $k < 0$

2. Horizontal Shifts

- $f(x - h)$ shifts the graph right if $h > 0$ and left if $h < 0$

3. Reflections

- $-f(x)$ reflects the graph over the x-axis - $f(-x)$ reflects the graph over the y-axis

4. Vertical Stretch/Compression

- $a \cdot f(x)$ stretches the graph vertically if $|a| > 1$ and compresses if $0 < |a| < 1$

5. Horizontal Stretch/Compression

- $f(bx)$ compresses horizontally if $|b| > 1$ and stretches if $0 < |b| < 1$

6. Combined Transformations

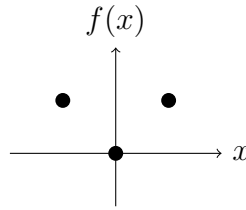
- Multiple transformations can occur together: $y = af(b(x - h)) + k$

7. Ways to Represent a Function

- Table

x	-1	0	1
$f(x)$	1	0	1

- Graph



- Equation

$$y = f(x)$$

Walkthrough of Example Questions

Example 1: Graph $y = f(x) + 3$ if $f(x) = x^2$.

Solution:

- Start with the parent function $y = x^2$.
- Apply a vertical shift up 3 units: add +3 to all y -values.
- The vertex moves from $(0, 0)$ to $(0, 3)$.

Example 2: Graph $y = -f(x)$ if $f(x) = \sqrt{x}$.

Solution:

- Start with the parent function $y = \sqrt{x}$.
- Reflect over the x -axis: multiply all y -values by -1 .
- Point $(4, 2)$ becomes $(4, -2)$; point $(1, 1)$ becomes $(1, -1)$.

Example 3: Graph $y = 2f(0.5(x - 2)) + 1$ if $f(x) = x^2$.

Solution:

- Horizontal stretch by factor $1/0.5 = 2$: replace x with $0.5x$ stretches the graph wider.
- Shift right 2 units: replace x with $(x - 2)$.
- Vertical stretch by factor 2: multiply all y -values by 2.
- Vertical shift up 1: add +1 to all y -values.
- The vertex moves from $(0, 0)$ to $(2, 1)$.

Worksheet: Practice Questions

1. Graph $y = x^2 + 4$. Identify the vertex.
2. Graph $y = -(x - 1)^2$. Identify the vertex and axis of symmetry.
3. Graph $y = 3\sqrt{x}$. Describe the transformation relative to $y = \sqrt{x}$.
4. Graph $y = f(-x)$ if $f(x) = x^3$. Describe the reflection.
5. Graph $y = 0.5(x + 2)^2 - 1$. Identify the vertex and describe the transformation.
6. If $y = 2f(3(x - 1)) + 5$ and $f(x) = x^2$, describe all transformations applied to the parent function.
7. A function $f(x) = x^2$ is transformed to $g(x) = -2f(x - 3) + 1$. Find the vertex and explain the steps.

Answer Key

Q1: $y = x^2 + 4$ Vertex: (0,4) — vertical shift up 4 units from parent function (0,0)

Q2: $y = -(x - 1)^2$ Vertex: (1,0) — shift right 1 unit, reflected over x-axis Axis of symmetry: $x = 1$

Q3: $y = 3\sqrt{x}$ Vertical stretch by factor 3. Parent function $y = \sqrt{x}$ is stretched taller.

Q4: $y = f(-x) = (-x)^3 = -x^3$ Reflected over y-axis. Point (2,8) maps to (-2,-8), point (-1,-1) maps to (1,1)

Q5: $y = 0.5(x + 2)^2 - 1$ Vertex: (-2,-1) — horizontal shift left 2 units, vertical shift down 1 unit, vertical compression by 0.5

Q6: $y = 2f(3(x - 1)) + 5$ - Horizontal compression by factor 1/3 - Shift right 1 unit - Vertical stretch by factor 2 - Vertical shift up 5 Vertex moves from (0,0) to (1,5)

Q7: $g(x) = -2f(x - 3) + 1$ - Shift right 3 units: vertex from $(0,0) \rightarrow (3,0)$ - Vertical stretch by factor 2: vertex remains $(3,0)$ - Reflection over x-axis: still $(3,0)$ - Vertical shift up 1: final vertex $(3,1)$