

Name: _____ Date: _____ Mark: _____/30

Pre-Calculus 12 Mock Exam

Multiple Choice (20 marks)

Instructions: Choose the best answer for each question. Each question is worth 1 mark.

1. If $f(x) = 2x^2 - 5x + 3$, what is $f(4)$?
A) 15 B) 19 C) 23 D) 27
2. Solve for x : $\log_3(2x - 1) = 4$. A) 40
B) 41 C) 81 D) 162
3. The horizontal asymptote of $\frac{3x^2+4x-5}{6x^2-x+2}$ is: A) $y = 0$ B) $y = \frac{3}{6}$ C) $y = 3$
D) $y = 6$
4. Which of the following is the sum formula for $\sin(A + B)$? A) $\sin A \cos B + \cos A \sin B$ B) $\sin A \cos B - \cos A \sin B$
C) $\cos A \cos B - \sin A \sin B$ D) $\cos A \cos B + \sin A \sin B$
5. The 5th term of the sequence $a_n = 3(2)^{n-1}$ is: A) 24 B) 48 C) 96 D) 192
6. If θ is in quadrant II and $\sin \theta = \frac{3}{5}$, then $\cos \theta$ is: A) $\frac{4}{5}$ B) $-\frac{4}{5}$ C) $\frac{5}{4}$ D) $-\frac{5}{4}$
7. The inverse of $f(x) = 5x - 2$ is: A) $f^{-1}(x) = \frac{x+2}{5}$ B) $f^{-1}(x) = \frac{x-2}{5}$ C) $f^{-1}(x) = 5x + 2$
D) $f^{-1}(x) = \frac{5}{x+2}$
8. Solve: $2^{x+1} = 32$. A) 4 B) 5 C) 6
D) 7
9. The remainder when $x^3 - 4x^2 + 5x - 2$ is divided by $(x - 2)$ is: A) 0 B) 1 C) 2
D) 3
10. Which conic has equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$?
A) Ellipse B) Circle C) Parabola
D) Hyperbola
11. The period of $y = 3 \sin(4x)$ is: A) $\pi/2$
B) π C) 2π D) 4π
12. $\cos(2\theta)$ equals: A) $\cos^2 \theta - \sin^2 \theta$ B) $2 \cos^2 \theta - 1$
C) $1 - 2 \sin^2 \theta$ D) All of the above
13. Which term is independent of x in the expansion of $(x^2 + \frac{3}{x})^6$? A) 405 B) 243
C) 729 D) 81
14. If $f(x) = \sqrt{x+4}$, the domain is: A) $x > 4$ B) $x \geq 4$ C) $x \geq -4$ D) All real numbers
15. Convert 135° to radians. A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$
C) $\frac{5\pi}{6}$ D) $\frac{7\pi}{6}$
16. Which describes an exponential decay?
A) Base > 1 B) Base $= 1$ C) $0 < \text{base} < 1$ D) Base < 0
17. The eccentricity of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is: A) $\frac{3}{5}$
B) $\frac{4}{5}$ C) $\frac{5}{3}$ D) $\frac{5}{4}$
18. $\tan(225^\circ)$ equals: A) 1 B) -1 C) $\sqrt{3}$
D) $-\sqrt{3}$
19. If $S_n = \frac{n}{2}(5n + 1)$, the first term is: A) 3
B) 5 C) 6 D) 7

20. Which of the following is a vertical asymptote of $y = \frac{x^2-4}{x^2-x-6}$? A) $x = 2$ B) $x = 3$ C) $x = -3$ D) B and C

Written Response (10 marks)

Instructions: Show all work. Each question is worth 1 mark.

21. Solve for x : $\log(x+2) + \log(x-1) = 1$.
22. The n^{th} term of a sequence is $a_n = 7 - 3n$. Find S_{15} .
23. Find the equation of the parabola with vertex $(2, -3)$ that passes through $(4, 5)$.
24. A ferris wheel has a radius of 15 m and completes one revolution every 40 seconds. Write a sine function to model the height above the ground if the lowest point is 2 m above the ground.
25. Expand and simplify: $(2x - \frac{1}{x^2})^4$.
26. Solve: $3\sin(2x) = 1$ for $0 \leq x < 2\pi$.
27. Using the Binomial Theorem, find the coefficient of x^5 in $(x+2)^9$.
28. Determine the range of $y = \frac{x^2+3}{x^2-1}$.
29. Use synthetic division to divide $x^3 - 6x^2 + 11x - 6$ by $(x-1)$.
30. A population grows according to $P(t) = 500(1.04)^t$. Find the time for the population to double.

Answer Key

Multiple Choice

- 1) B 2) B 3) B 4) A 5) C 6) B 7) B 8) B 9) C 10) D
11) A 12) D 13) A 14) C 15) B 16) C 17) B 18) A 19) B 20) D

Written Response (Sample Solutions)

- 21) $x = 3$ 22) $S_{15} = -255$ 23) $y = 2(x - 2)^2 - 3$ 24) $h(t) = 15 \sin\left(\frac{\pi}{20}t - \frac{\pi}{2}\right) + 17$
25) $16x^4 - 32x + 24x^{-2} - 8x^{-5} + x^{-8}$ 26) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
27) Coefficient = 8064 28) Range: $(-\infty, 1) \cup (3, \infty)$ 29) Quotient: $x^2 - 5x + 6$ remainder 0
30) $t \approx 17.67$ years

Deep Walkthrough

Q1: Evaluate $f(2)$ for $f(x) = 3x^2 - 5x + 2$. Calculate:

$$3(2)^2 - 5(2) + 2 = 3 \times 4 - 10 + 2 = 12 - 10 + 2 = 4.$$

Answer: B (4). Tip: Follow order of operations carefully.

Q2: Solve $2^{x+1} = 16$. Rewrite 16 as a power of 2: $16 = 2^4$. Set exponents equal:

$$x + 1 = 4 \implies x = 3.$$

Answer: B (3). Common mistake: Forgetting to rewrite 16 as 2^4 .

Q3: Simplify $\log_2(8) + \log_2(4)$. Use log product rule: $\log_a M + \log_a N = \log_a(MN)$. Calculate inside log: $8 \times 4 = 32$. So the expression is $\log_2(32)$. *Answer: A.*

Q4: Sum of sequence: $S_n = 3n^2 + 2n$. Find 5th term. Recall $t_n = S_n - S_{n-1}$. Calculate:

$$S_5 = 3(25) + 2(5) = 75 + 10 = 85,$$

$$S_4 = 3(16) + 2(4) = 48 + 8 = 56,$$

$$t_5 = S_5 - S_4 = 85 - 56 = 29.$$

Answer: C (29).

Q5: Period of $y = 2 \sin(3x)$. Period of $\sin(kx) = \frac{2\pi}{k}$. Here, $k = 3$, so

$$\text{Period} = \frac{2\pi}{3}.$$

Answer: B.

Q6: Find a if $(x + 2)$ is a factor of $x^3 + ax^2 - 4x - 8$. Use Factor Theorem: substitute $x = -2$ and set equal to zero:

$$(-2)^3 + a(-2)^2 - 4(-2) - 8 = 0,$$

$$\begin{aligned} -8 + 4a + 8 - 8 &= 0, \\ 4a - 8 &= 0 \implies 4a = 8 \implies a = 2. \end{aligned}$$

Answer: A (2).

Q7: Horizontal asymptote of $\frac{2x^2+3}{x^2-5}$. Compare degrees: numerator and denominator both degree 2. Horizontal asymptote = ratio of leading coefficients = $\frac{2}{1} = 2$. *Answer: C ($y = 2$).*

Q8: Eccentricity of a circle. A circle has eccentricity $e = 0$. *Answer: A (0).*

Q9: Simplify $\sin^2 \theta + \cos^2 \theta$. By Pythagorean identity, this equals 1. *Answer: B (1).*

Q10: Solve $\log_5(125) = x$. Rewrite $125 = 5^3$. So, $x = 3$. *Answer: B (3).*

Q11: Domain of $f(x) = \sqrt{x+4}$. Inside root must be ≥ 0 :

$$x + 4 \geq 0 \implies x \geq -4.$$

Answer: B ($x \geq -4$).

Q12: Vertex of $y = 2(x-3)^2 + 5$. Vertex form is $(h, k) = (3, 5)$. *Answer: A (3, 5).*

Q13: Inverse of $y = 3x - 7$. Switch x and y , solve for y :

$$x = 3y - 7 \implies 3y = x + 7 \implies y = \frac{x+7}{3}.$$

Answer: A.

Q14: Dot product $u \cdot v$ with $u = \langle 2, -1 \rangle$, $v = \langle -3, 4 \rangle$. Calculate:

$$2 \times (-3) + (-1) \times 4 = -6 - 4 = -10.$$

Answer: B (-10).

Q15: Graph shift of $y = |x - 2|$. Inside the absolute value is $x - 2$, which shifts the graph 2 units to the right. *Answer: B (2 units right).*

Q16: Solve $\sin \theta = \frac{\sqrt{3}}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. Sine equals $\frac{\sqrt{3}}{2}$ at 60° and 120° . *Answer: A ($60^\circ, 120^\circ$).*

Q17: 4th term of geometric sequence with $r = 3$ and $t_1 = 5$. Use $t_n = t_1 r^{n-1}$:

$$t_4 = 5 \times 3^3 = 5 \times 27 = 135.$$

Answer: B (135).

Q18: Horizontal asymptote of $y = e^x$. As $x \rightarrow -\infty$, $y \rightarrow 0$. So horizontal asymptote is $y = 0$. *Answer: A.*

Q19: Derivative of $f(x) = 5x^3$. Use power rule:

$$f'(x) = 5 \times 3x^2 = 15x^2.$$

Answer: A.

Q20: Domain of $\sqrt{x+4}$ is $x \geq -4$. *Answer: B.* (Repeated for clarity.)

Q21: Solve $3^{2x-1} = 27$. Rewrite $27 = 3^3$. Equate exponents:

$$2x - 1 = 3 \implies 2x = 4 \implies x = 2.$$

Q22: Sum of first 12 terms of arithmetic sequence with $t_1 = 4$, $d = 7$. Use formula:

$$S_n = \frac{n}{2}[2t_1 + (n-1)d] = \frac{12}{2}[2 \times 4 + 11 \times 7] = 6[8 + 77] = 6 \times 85 = 510.$$

Q23: Equation of parabola with vertex $(2, -3)$ passing through $(4, 5)$. Form: $y = a(x - h)^2 + k$. Substitute $h = 2$, $k = -3$:

$$5 = a(4 - 2)^2 - 3 \implies 5 = 4a - 3 \implies 4a = 8 \implies a = 2.$$

Equation:

$$y = 2(x - 2)^2 - 3.$$

Q24: Expand $(2x - 3)^3$. Use binomial expansion:

$$(2x)^3 - 3 \times (2x)^2 \times 3 + 3 \times (2x) \times 3^2 - 3^3 = 8x^3 - 36x^2 + 54x - 27.$$

Q25: Factor $x^4 - 81$. Difference of squares:

$$x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9).$$

Further factor $x^2 - 9$:

$$(x - 3)(x + 3)(x^2 + 9).$$

Q26: Amplitude and period of $y = -4 \cos\left(\frac{x}{2}\right)$. Amplitude = absolute value of coefficient = 4. Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$.

Q27: Solve $2 \cos \theta - 1 = 0$.

$$2 \cos \theta = 1 \implies \cos \theta = \frac{1}{2}.$$

Cosine equals $\frac{1}{2}$ at $\theta = 60^\circ$ and 300° .

Q28: First 4 terms of $t_n = 5(2)^{n-1}$. Calculate:

$$t_1 = 5, \quad t_2 = 10, \quad t_3 = 20, \quad t_4 = 40.$$

Q29: Domain and range of $y = \sqrt{4 - x}$. Domain: inside root $\geq 0 \implies 4 - x \geq 0 \implies x \leq 4$. Range: square root outputs ≥ 0 .

$$\text{Domain : } x \leq 4, \quad \text{Range : } y \geq 0.$$

Q30: Simplify $\frac{x^2-9}{x^2-x-6}$. Factor numerator and denominator:

$$\frac{(x-3)(x+3)}{(x-3)(x+2)}.$$

Cancel $(x-3)$:

$$\frac{x+3}{x+2}.$$

Domain note: $x \neq 3, -2$.