

Angles and the Coordinate Plane

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Notes

What Is an Angle?

An **angle** is formed by two rays that share a common endpoint called the **vertex**. In trigonometry, we place angles in **standard position**: the vertex sits at the origin, and one ray (the **initial side**) points along the positive x -axis. The other ray (the **terminal side**) is where the angle finishes after rotating.

- A **positive angle** is formed by rotating **counterclockwise**.
- A **negative angle** is formed by rotating **clockwise**.

The Four Quadrants

The coordinate plane is divided into four regions called **quadrants**, numbered counterclockwise starting from the upper-right:

- **Quadrant I:** $x > 0$ and $y > 0$ (terminal side between 0° and 90°)
- **Quadrant II:** $x < 0$ and $y > 0$ (terminal side between 90° and 180°)
- **Quadrant III:** $x < 0$ and $y < 0$ (terminal side between 180° and 270°)
- **Quadrant IV:** $x > 0$ and $y < 0$ (terminal side between 270° and 360°)

An angle whose terminal side lies exactly on an axis (at 0° , 90° , 180° , 270° , or 360°) is called a **quadrantal angle**.

Coterminal Angles

Two angles are **coterminal** if they share the same terminal side. To find a coterminal angle, add or subtract one full rotation of 360° :

$$\theta_{\text{coterminal}} = \theta \pm 360^\circ \cdot n, \quad n = 1, 2, 3, \dots$$

Reference Angles

The **reference angle** is the positive acute angle (between 0° and 90°) formed between the terminal side and the nearest part of the x -axis. Use the formula that matches the quadrant the terminal side is in:

$$\text{Quadrant I: } \theta_{\text{ref}} = \theta$$

$$\text{Quadrant II: } \theta_{\text{ref}} = 180^\circ - \theta$$

$$\text{Quadrant III: } \theta_{\text{ref}} = \theta - 180^\circ$$

$$\text{Quadrant IV: } \theta_{\text{ref}} = 360^\circ - \theta$$

If $\theta > 360^\circ$ or $\theta < 0^\circ$, reduce it to the interval $[0^\circ, 360^\circ)$ first, then apply the appropriate formula above.

Pro Tip

- To identify the quadrant of an angle larger than 360° , keep subtracting 360° until you land between 0° and 360° , then check the interval.
- A reference angle is **always positive** and **always between 0° and 90°** . If your answer falls outside this range, recheck which formula you applied.
- Coterminal angles have the **exact same trigonometric values**. This becomes very useful later when solving trig equations.
- Angles in standard position always start measuring from the **positive x -axis**. Do not measure from the y -axis or from a different starting direction.

Examples

Example 1: Identify the quadrant of $\theta = 150^\circ$.

Step 1: Check which interval 150° belongs to.

$$90^\circ < 150^\circ < 180^\circ$$

Step 2: State the quadrant. Since 150° is between 90° and 180° , the terminal side lies in **Quadrant II**.

Example 2: Find one positive and one negative angle coterminal with $\theta = 75^\circ$.

Step 1: Add 360° to find a positive coterminal angle.

$$75^\circ + 360^\circ = 435^\circ$$

Step 2: Subtract 360° to find a negative coterminal angle.

$$75^\circ - 360^\circ = -285^\circ$$

Both 435° and -285° are coterminal with 75° .

Example 3: Find the reference angle for $\theta = 210^\circ$.

Step 1: Identify the quadrant.

$$180^\circ < 210^\circ < 270^\circ \quad \Rightarrow \quad \text{Quadrant III}$$

Step 2: Apply the Quadrant III formula.

$$\theta_{\text{ref}} = \theta - 180^\circ = 210^\circ - 180^\circ$$

$$\theta_{\text{ref}} = 30^\circ$$

Example 4: Find the reference angle for $\theta = 300^\circ$.

Step 1: Identify the quadrant.

$$270^\circ < 300^\circ < 360^\circ \Rightarrow \text{Quadrant IV}$$

Step 2: Apply the Quadrant IV formula.

$$\theta_{\text{ref}} = 360^\circ - \theta = 360^\circ - 300^\circ$$

$$\theta_{\text{ref}} = 60^\circ$$

Example 5: Find the reference angle for $\theta = 480^\circ$.

Step 1: Reduce the angle since it is greater than 360° .

$$480^\circ - 360^\circ = 120^\circ$$

Step 2: Identify the quadrant of 120° .

$$90^\circ < 120^\circ < 180^\circ \Rightarrow \text{Quadrant II}$$

Step 3: Apply the Quadrant II formula.

$$\theta_{\text{ref}} = 180^\circ - 120^\circ = 60^\circ$$

Common Mistakes

- **Not placing the angle in standard position first.** Always start the initial side on the positive x -axis before identifying the quadrant or reference angle.
- **Using the wrong reference angle formula.** Each quadrant has its own formula. Mixing up the Quadrant II and Quadrant III formulas is very common — double-check which one applies before computing.
- **Getting a negative reference angle.** Reference angles are always positive. If your result is negative, you subtracted in the wrong order.
- **Confusing coterminal angles with supplementary angles.** Coterminal angles differ by multiples of 360° , not 180° .
- **Skipping the reduction step.** If $\theta > 360^\circ$ or $\theta < 0^\circ$, always reduce to $[0^\circ, 360^\circ)$ before finding the reference angle.

Worksheet

1. Identify the quadrant (or axis) in which the terminal side of each angle lies.

(a) $\theta = 45^\circ$

(b) $\theta = 120^\circ$

- (c) $\theta = 250^\circ$
- (d) $\theta = 330^\circ$
- (e) $\theta = 180^\circ$

2. Find one positive and one negative angle coterminal with each angle.

- (a) $\theta = 50^\circ$
- (b) $\theta = 200^\circ$
- (c) $\theta = -30^\circ$
- (d) $\theta = 400^\circ$

3. Find the reference angle for each angle.

- (a) $\theta = 135^\circ$
- (b) $\theta = 220^\circ$
- (c) $\theta = 315^\circ$
- (d) $\theta = 480^\circ$
- (e) $\theta = -60^\circ$

4. An angle in standard position has its terminal side in Quadrant III and a reference angle of 40° . What is the measure of the original angle?

5. Find the smallest positive angle coterminal with $\theta = -110^\circ$.

Answer Key

1. Identifying the quadrant of each angle.

(a) $\theta = 45^\circ$

Step 1: Check the interval.

$$0^\circ < 45^\circ < 90^\circ$$

The terminal side lies in Quadrant I.

Quadrant I

(b) $\theta = 120^\circ$

Step 1: Check the interval.

$$90^\circ < 120^\circ < 180^\circ$$

The terminal side lies in Quadrant II.

Quadrant II

(c) $\theta = 250^\circ$

Step 1: Check the interval.

$$180^\circ < 250^\circ < 270^\circ$$

The terminal side lies in Quadrant III.

Quadrant III

(d) $\theta = 330^\circ$

Step 1: Check the interval.

$$270^\circ < 330^\circ < 360^\circ$$

The terminal side lies in Quadrant IV.

Quadrant IV

(e) $\theta = 180^\circ$

Step 1: 180° lands exactly on the negative x -axis. This does not belong to any quadrant.

Negative x -axis (quadrantal angle)

2. Coterminal angles.

(a) $\theta = 50^\circ$

Step 1: Add 360° .

$$50^\circ + 360^\circ = 410^\circ$$

Step 2: Subtract 360° .

$$50^\circ - 360^\circ = -310^\circ$$

410° and -310°

(b) $\theta = 200^\circ$

Step 1: Add 360° .

$$200^\circ + 360^\circ = 560^\circ$$

Step 2: Subtract 360° .

$$200^\circ - 360^\circ = -160^\circ$$

$$\boxed{560^\circ \text{ and } -160^\circ}$$

(c) $\theta = -30^\circ$

Step 1: Add 360° to find a positive coterminal angle.

$$-30^\circ + 360^\circ = 330^\circ$$

Step 2: Subtract 360° to find another negative coterminal angle.

$$-30^\circ - 360^\circ = -390^\circ$$

$$\boxed{330^\circ \text{ and } -390^\circ}$$

(d) $\theta = 400^\circ$

Step 1: 400° exceeds 360° . Subtract 360° to find the equivalent positive angle.

$$400^\circ - 360^\circ = 40^\circ$$

Step 2: Subtract 360° for a negative coterminal angle.

$$40^\circ - 360^\circ = -320^\circ$$

$$\boxed{40^\circ \text{ and } -320^\circ}$$

3. Reference angles.

(a) $\theta = 135^\circ$

Step 1: $90^\circ < 135^\circ < 180^\circ$, so the terminal side is in Quadrant II.

Step 2: Apply the Quadrant II formula.

$$\theta_{\text{ref}} = 180^\circ - 135^\circ = 45^\circ$$

$$\boxed{45^\circ}$$

(b) $\theta = 220^\circ$

Step 1: $180^\circ < 220^\circ < 270^\circ$, so the terminal side is in Quadrant III.

Step 2: Apply the Quadrant III formula.

$$\theta_{\text{ref}} = 220^\circ - 180^\circ = 40^\circ$$

$$\boxed{40^\circ}$$

(c) $\theta = 315^\circ$

Step 1: $270^\circ < 315^\circ < 360^\circ$, so the terminal side is in Quadrant IV.

Step 2: Apply the Quadrant IV formula.

$$\theta_{\text{ref}} = 360^\circ - 315^\circ = 45^\circ$$

$$\boxed{45^\circ}$$

(d) $\theta = 480^\circ$

Step 1: Reduce the angle.

$$480^\circ - 360^\circ = 120^\circ$$

Step 2: $90^\circ < 120^\circ < 180^\circ$, so the terminal side is in Quadrant II.

Step 3: Apply the Quadrant II formula.

$$\theta_{\text{ref}} = 180^\circ - 120^\circ = 60^\circ$$

$$\boxed{60^\circ}$$

(e) $\theta = -60^\circ$

Step 1: Add 360° to make the angle positive.

$$-60^\circ + 360^\circ = 300^\circ$$

Step 2: $270^\circ < 300^\circ < 360^\circ$, so the terminal side is in Quadrant IV.

Step 3: Apply the Quadrant IV formula.

$$\theta_{\text{ref}} = 360^\circ - 300^\circ = 60^\circ$$

$$\boxed{60^\circ}$$

4. Quadrant III, reference angle $= 40^\circ$.

Step 1: In Quadrant III, the angle is 180° plus the reference angle.

$$\theta = 180^\circ + 40^\circ = 220^\circ$$

$$\boxed{220^\circ}$$

5. Smallest positive coterminal angle for $\theta = -110^\circ$.

Step 1: Add 360° .

$$-110^\circ + 360^\circ = 250^\circ$$

Step 2: Check that 250° is between 0° and 360° . It is.

$$\boxed{250^\circ}$$

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