Graphing Sine and Cosine Functions
ApexMath

Notes

The Parent Functions

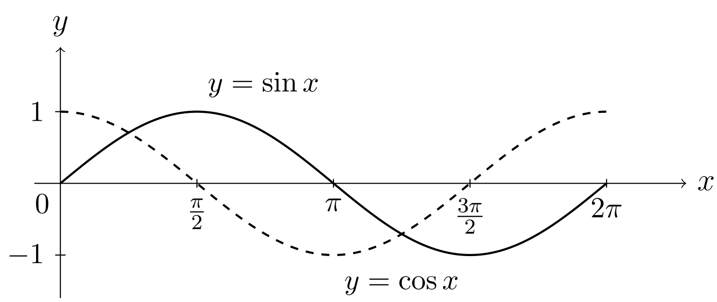
The two most fundamental trig graphs are $y = \sin x$ and $y = \cos x$. Both have the same basic wave shape but start at different points.

Key features of $y = \sin x$:

- Starts at (0, 0), rises to a maximum of 1 at $x = \dfrac{\pi}{2}$
- Returns to 0 at $x = \pi$, drops to -1 at $x = \dfrac{3\pi}{2}$, returns to 0 at $x = 2\pi$
- Period $= 2\pi$

Key features of $y = \cos x$:

- Starts at (0, 1), drops to 0 at $x = \dfrac{\pi}{2}$
- Reaches -1 at $x = \pi$, returns to 0 at $x = \dfrac{3\pi}{2}$, returns to 1 at $x = 2\pi$
- Period $= 2\pi$



The General Form

Any transformed sine or cosine function is written as:

$$y = A\sin(Bx - C) + D \qquad \text{or} \qquad y = A\cos(Bx - C) + D$$

Each letter controls one specific transformation:

- $|A|$ = **Amplitude** — distance from the midline to the maximum
- $\dfrac{2\pi}{|B|}$ = **Period** — length of one full cycle

- $\dfrac{C}{B}$ = **Phase Shift** — horizontal shift (positive = right, negative = left)
- D = **Vertical Shift** — midline moves to $y = D$

If A is negative, the graph is **reflected** (flipped upside down).
Maximum and Minimum Values

$$\text{Maximum} = D + |A| \qquad \text{Minimum} = D - |A|$$

Step-by-Step Graphing Process

1. Identify A, B, C, D.
2. Calculate amplitude, period, phase shift, and midline.
3. Divide the period into 4 equal parts to find the spacing between key points.
4. Plot the five key points across one full period and sketch the wave.

Pro Tip

- The **midline** is $y = D$. The graph oscillates equally above and below it. Maximum is at $D + |A|$ and minimum is at $D - |A|$.
- **Always factor out B before reading the phase shift.** For $y = \sin(2x - \pi)$, factor as $y = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$ to see the phase shift is $+\frac{\pi}{2}$ (right), not π.
- **A larger $|B|$ compresses the graph** — the period gets shorter. A smaller $|B|$ stretches the graph. Always divide 2π by $|B|$, never multiply.
- **If A is negative**, the wave starts by going down from the midline. The maximum and minimum positions swap compared to the positive case.

Examples

Example 1: State the amplitude, period, phase shift, and vertical shift of $y = 3\sin(2x) + 1$.
Step 1: Identify A, B, C, D.

$$A = 3, \quad B = 2, \quad C = 0, \quad D = 1$$

Step 2: Amplitude.

$$|A| = 3$$

Step 3: Period.

$$P = \frac{2\pi}{|B|} = \frac{2\pi}{2} = \pi$$

Step 4: Phase shift. $\dfrac{C}{B} = \dfrac{0}{2} = 0$ (no horizontal shift).
Step 5: Vertical shift. Midline at $y = 1$. Maximum $= 1 + 3 = 4$. Minimum $= 1 - 3 = -2$.

Example 2: State the amplitude, period, phase shift, and vertical shift of $y = -2\cos(3x - \pi) + 4$.
Step 1: Identify A, B, C, D.

$$A = -2, \quad B = 3, \quad C = \pi, \quad D = 4$$

Step 2: Amplitude. $|A| = 2$. The negative sign means the graph is reflected.
Step 3: Period.

$$P = \frac{2\pi}{3}$$

Step 4: Phase shift. Factor out $B = 3$.

$$3x - \pi = 3\left(x - \frac{\pi}{3}\right) \quad \Rightarrow \quad \text{Phase shift} = +\frac{\pi}{3} \text{ (right)}$$

Step 5: Vertical shift. Midline at $y = 4$. Maximum $= 6$. Minimum $= 2$.

Example 3: Find the five key points for one period of $y = 2\sin(x) - 1$.
Step 1: Parameters. $A = 2$, $B = 1$, $C = 0$, $D = -1$. Period $= 2\pi$, midline $y = -1$.
Step 2: Key x-values are spaced $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$ apart.
Step 3: Evaluate at each key x.

x	$y = 2\sin(x) - 1$
0	$2(0) - 1 = -1$
$\frac{\pi}{2}$	$2(1) - 1 = 1$
π	$2(0) - 1 = -1$
$\frac{3\pi}{2}$	$2(-1) - 1 = -3$
2π	$2(0) - 1 = -1$

Common Mistakes

- **Confusing amplitude with the maximum value.** Amplitude is $|A|$, the distance from the midline to the peak. The actual maximum is $D + |A|$.

- **Multiplying instead of dividing for the period.** The period is $\dfrac{2\pi}{|B|}$. A larger $|B|$ makes the period *shorter*, not longer.

- **Not factoring out B before reading the phase shift.** In $y = \sin(2x - \pi)$ the phase shift is $\dfrac{\pi}{2}$, not π. Factor first, every time.

- **Ignoring the reflection when $A < 0$.** A negative A flips the wave. The first movement from the midline goes *down*, not up.

Worksheet

1. State the amplitude, period, phase shift, and vertical shift for each function.
   (a) $y = 4\sin(x)$
   (b) $y = \cos(3x) - 2$
   (c) $y = -3\sin(2x + \pi)$
   (d) $y = \frac{1}{2}\cos\left(x - \frac{\pi}{4}\right) + 3$

2. Find the maximum and minimum values of each function.
   (a) $y = 5\sin(x) + 2$
   (b) $y = -4\cos(x) - 1$
   (c) $y = 3\sin(2x - \pi) + 6$

3. For $y = 2\cos\left(\frac{x}{2}\right) + 1$:
   (a) Find the amplitude, period, phase shift, and vertical shift.
   (b) Find the five key points for one full period starting at $x = 0$.

4. Write the equation of a sine function with amplitude 3, period π, phase shift $\frac{\pi}{4}$ to the right, and no vertical shift.

5. A sine function has a maximum value of 7 and a minimum value of 1. Find the amplitude and the vertical shift.

Answer Key

1. Parameters.

(a) $y = 4\sin(x)$: $A = 4,\ B = 1,\ C = 0,\ D = 0$

$$\text{Amplitude} = 4, \qquad P = 2\pi, \qquad \text{Phase shift} = 0, \qquad \text{Vertical shift} = 0$$

$$\boxed{\text{Amplitude } 4, \quad \text{Period } 2\pi, \quad \text{No phase shift}, \quad \text{No vertical shift}}$$

(b) $y = \cos(3x) - 2$: $A = 1,\ B = 3,\ C = 0,\ D = -2$

$$\text{Amplitude} = 1, \qquad P = \frac{2\pi}{3}, \qquad \text{Phase shift} = 0, \qquad \text{Vertical shift} = -2$$

$$\boxed{\text{Amplitude } 1, \quad \text{Period } \frac{2\pi}{3}, \quad \text{No phase shift}, \quad \text{Down } 2}$$

(c) $y = -3\sin(2x + \pi)$: $A = -3,\ B = 2,\ C = -\pi,\ D = 0$
Step 1: Factor out $B = 2$ to find the phase shift.

$$2x + \pi = 2\left(x + \frac{\pi}{2}\right) \quad \Rightarrow \quad \text{Phase shift} = -\frac{\pi}{2} \text{ (left)}$$

$$\text{Amplitude} = 3 \text{ (reflected)}, \qquad P = \pi$$

$$\boxed{\text{Amplitude } 3 \text{ (reflected)}, \quad \text{Period } \pi, \quad \text{Phase shift } \frac{\pi}{2} \text{ left}}$$

(d) $y = \dfrac{1}{2}\cos\left(x - \dfrac{\pi}{4}\right) + 3$: $A = \frac{1}{2},\ B = 1,\ C = \frac{\pi}{4},\ D = 3$

$$\text{Amplitude} = \frac{1}{2}, \qquad P = 2\pi, \qquad \text{Phase shift} = \frac{\pi}{4} \text{ (right)}, \qquad \text{Vertical shift} = 3$$

$$\boxed{\text{Amplitude } \frac{1}{2}, \quad \text{Period } 2\pi, \quad \text{Phase shift } \frac{\pi}{4} \text{ right}, \quad \text{Up } 3}$$

2. Maximum and minimum values.

(a) $y = 5\sin(x) + 2$: $|A| = 5,\ D = 2$

$$\text{Max} = 2 + 5 = 7 \qquad \text{Min} = 2 - 5 = -3$$

$$\boxed{\text{Max} = 7, \qquad \text{Min} = -3}$$

(b) $y = -4\cos(x) - 1$: $|A| = 4,\ D = -1$

$$\text{Max} = -1 + 4 = 3 \qquad \text{Min} = -1 - 4 = -5$$

$$\boxed{\text{Max} = 3, \qquad \text{Min} = -5}$$

(c) $y = 3\sin(2x - \pi) + 6$: $|A| = 3,\ D = 6$

$$\text{Max} = 6 + 3 = 9 \qquad \text{Min} = 6 - 3 = 3$$

$$\boxed{\text{Max} = 9, \qquad \text{Min} = 3}$$

3. $y = 2\cos\left(\frac{x}{2}\right) + 1$

(a) $A = 2,\ B = \frac{1}{2},\ C = 0,\ D = 1$

$$\text{Amplitude} = 2, \qquad P = \frac{2\pi}{1/2} = 4\pi, \qquad \text{Phase shift} = 0, \qquad \text{Vertical shift} = 1$$

$$\boxed{\text{Amplitude } 2, \quad \text{Period } 4\pi, \quad \text{No phase shift}, \quad \text{Up } 1}$$

(b) Key points are spaced $\frac{4\pi}{4} = \pi$ apart.

x	$y = 2\cos\left(\frac{x}{2}\right) + 1$
0	$2(1) + 1 = 3$
π	$2(0) + 1 = 1$
2π	$2(-1) + 1 = -1$
3π	$2(0) + 1 = 1$
4π	$2(1) + 1 = 3$

$$\boxed{(0,\, 3), \quad (\pi,\, 1), \quad (2\pi,\, -1), \quad (3\pi,\, 1), \quad (4\pi,\, 3)}$$

4. Amplitude $= 3$, period $= \pi$, phase shift $= \frac{\pi}{4}$ right, $D = 0$.
Step 1: Use the period to find B.

$$\frac{2\pi}{B} = \pi \quad \Rightarrow \quad B = 2$$

Step 2: Use the phase shift to find C.

$$\frac{C}{B} = \frac{\pi}{4} \quad \Rightarrow \quad C = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$$

Step 3: Write the equation.

$$y = 3\sin\left(2x - \frac{\pi}{2}\right)$$

$$\boxed{y = 3\sin\left(2x - \frac{\pi}{2}\right)}$$

5. Maximum $= 7$, minimum $= 1$.

Step 1: Amplitude is half the total range.

$$|A| = \frac{\text{Max} - \text{Min}}{2} = \frac{7-1}{2} = 3$$

Step 2: Vertical shift is the average of the max and min.

$$D = \frac{\text{Max} + \text{Min}}{2} = \frac{7+1}{2} = 4$$

$$\boxed{\text{Amplitude} = 3, \qquad \text{Vertical shift} = 4}$$

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