Graphing Tangent Functions
ApexMath

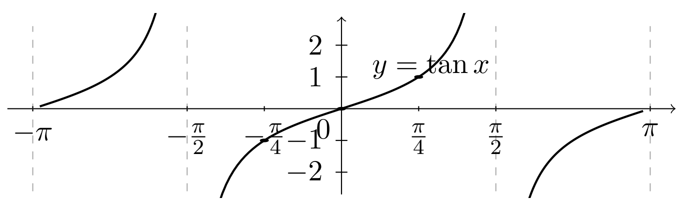
Notes

The Parent Tangent Function

Recall that $\tan x = \dfrac{\sin x}{\cos x}$. The tangent function is undefined wherever $\cos x = 0$, which happens at $x = \dfrac{\pi}{2} + n\pi$ for any integer n. At those x-values the graph has **vertical asymptotes** — the curve shoots off to $+\infty$ on one side and $-\infty$ on the other.

Key properties of $y = \tan x$:

- **Period** $= \pi$ (half the period of sine and cosine)
- **No amplitude** — the graph extends to $\pm\infty$ with no peak or trough
- **Vertical asymptotes** at $x = \dfrac{\pi}{2} + n\pi$
- Passes through $\left(-\dfrac{\pi}{4}, -1\right)$, $(0, 0)$, and $\left(\dfrac{\pi}{4}, 1\right)$
- Always **increasing** on each open interval between asymptotes



The General Form

$$y = A\tan(Bx - C) + D$$

- A = **vertical stretch** (steepness; there is no amplitude for tangent)
- $\dfrac{\pi}{|B|}$ = **Period**
- $\dfrac{C}{B}$ = **Phase Shift** (positive = right, negative = left)
- D = **Vertical Shift**

Finding the Vertical Asymptotes

Asymptotes occur where the argument equals $\dfrac{\pi}{2} + n\pi$. Solve:

$$Bx - C = \frac{\pi}{2} + n\pi \quad \Rightarrow \quad x = \frac{C}{B} + \frac{\pi}{2B} + \frac{n\pi}{B}$$

A quick shortcut: the two asymptotes nearest to the phase shift are $\frac{P}{2}$ away from it on each side.

Three Key Points per Period

For the parent $y = \tan x$, the three key points on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ are:

$$\left(-\frac{\pi}{4},\ -1\right), \qquad (0,\ 0), \qquad \left(\frac{\pi}{4},\ 1\right)$$

After transforming: shift x-values by the phase shift, multiply y-values by A, then add D.

Pro Tip

- **The period of tangent is π, not 2π.** Always use $\frac{\pi}{|B|}$, never $\frac{2\pi}{|B|}$. Using the wrong formula gives a graph twice as wide as it should be.
- **Draw the asymptotes first** as dashed vertical lines, then sketch the S-shaped curve between them. The curve can never touch or cross an asymptote.
- **Tangent has no maximum or minimum.** Do not apply $D \pm |A|$ for the range — tangent covers all real numbers within each period.
- **If A is negative**, the curve goes from upper-left to lower-right (decreasing) rather than lower-left to upper-right. It is reflected.

Examples

Example 1: State the period, asymptotes, and three key points of $y = \tan(2x)$.

Step 1: Identify $B = 2$.

Step 2: Period.

$$P = \frac{\pi}{|B|} = \frac{\pi}{2}$$

Step 3: Asymptotes. Solve $2x = \frac{\pi}{2} + n\pi$.

$$x = \frac{\pi}{4} + \frac{n\pi}{2}$$

Nearest to origin: $x = \frac{\pi}{4}$ and $x = -\frac{\pi}{4}$.

Step 4: Three key points. The centre key point is at $x = 0$. The outer two are at $x = \pm\frac{\pi}{8}$ (one quarter-period from the asymptotes).

$$\left(-\frac{\pi}{8},\ -1\right), \qquad (0,\ 0), \qquad \left(\frac{\pi}{8},\ 1\right)$$

Example 2: Find all parameters and asymptotes of $y = 2\tan\left(x - \frac{\pi}{4}\right) + 3$.

Step 1: Identify $A = 2$**,** $B = 1$**,** $C = \dfrac{\pi}{4}$**,** $D = 3$**.**
Step 2: Period. $P = \dfrac{\pi}{1} = \pi$.
Step 3: Phase shift. $\dfrac{C}{B} = \dfrac{\pi}{4}$ (right).
Step 4: Vertical shift. Midline at $y = 3$.
Step 5: Asymptotes. Parent asymptotes at $\pm\dfrac{\pi}{2}$, shifted right by $\dfrac{\pi}{4}$:

$$x = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} \qquad \text{and} \qquad x = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$$

Step 6: Three key points. Shift parent key points right $\dfrac{\pi}{4}$, then apply $y \to 2y + 3$:

Parent $(x,\, y)$	After shift right $\dfrac{\pi}{4}$	$y \to 2y + 3$
$\left(-\dfrac{\pi}{4},\, -1\right)$	$(0,\, -1)$	$(0,\ 1)$
$(0,\, 0)$	$\left(\dfrac{\pi}{4},\, 0\right)$	$\left(\dfrac{\pi}{4},\ 3\right)$
$\left(\dfrac{\pi}{4},\, 1\right)$	$\left(\dfrac{\pi}{2},\, 1\right)$	$\left(\dfrac{\pi}{2},\ 5\right)$

Example 3: Find the period, asymptotes, and three key points of $y = \tan\left(\dfrac{x}{2}\right)$.
Step 1: Identify $B = \dfrac{1}{2}$**.**
Step 2: Period.

$$P = \frac{\pi}{1/2} = 2\pi$$

Step 3: Asymptotes. Solve $\dfrac{x}{2} = \dfrac{\pi}{2} + n\pi$:

$$x = \pi + 2n\pi \quad \Rightarrow \quad \text{Nearest: } x = \pi \text{ and } x = -\pi$$

Step 4: Three key points. Period is 2π, so key points are $\dfrac{\pi}{2}$ from centre:

$$\left(-\frac{\pi}{2},\ -1\right), \qquad (0,\ 0), \qquad \left(\frac{\pi}{2},\ 1\right)$$

Common Mistakes

- **Using period 2π instead of π.** The period of tangent is $\dfrac{\pi}{|B|}$. Using the sine/cosine formula gives a graph twice as wide.
- **Drawing tangent like a sine wave.** Tangent has no peak or trough. Each branch is S-shaped and continues to infinity. It is not a bounded wave.

- **Forgetting to draw the asymptotes first.** Without asymptotes, it is nearly impossible to sketch a correct tangent graph. Mark them as dashed lines before drawing anything else.
- **Letting the curve cross the asymptote.** Each branch of the tangent curve stays strictly within its interval and can never touch an asymptote.

Worksheet

1. State the period, phase shift, vertical shift, and the two asymptotes nearest the origin for each function.

   (a) $y = \tan(3x)$

   (b) $y = \tan\left(x + \dfrac{\pi}{6}\right)$

   (c) $y = -\tan(x) + 2$

   (d) $y = 3\tan\left(2x - \dfrac{\pi}{2}\right)$

2. Find the three key points for one period of each function.

   (a) $y = \tan(x)$

   (b) $y = 2\tan(x)$

   (c) $y = \tan(2x)$

3. For $y = -2\tan\left(x - \dfrac{\pi}{4}\right) + 1$:

   (a) State the period, phase shift, and vertical shift.

   (b) Find the two asymptotes nearest the phase shift.

   (c) Find the three key points in the central period.

4. A tangent function has period $\dfrac{\pi}{3}$. What is the value of $|B|$?

5. Explain in your own words why $y = \tan x$ has no amplitude.

Answer Key

1. Parameters and asymptotes.

(a) $y = \tan(3x)$: $B = 3$

$$P = \frac{\pi}{3}, \qquad \text{No phase or vertical shift}$$

Asymptotes: solve $3x = \frac{\pi}{2}$ and $3x = -\frac{\pi}{2}$.

$$x = \frac{\pi}{6} \quad \text{and} \quad x = -\frac{\pi}{6}$$

$$\boxed{P = \frac{\pi}{3}, \quad \text{No shifts,} \quad \text{Asymptotes at } x = \pm\frac{\pi}{6}}$$

(b) $y = \tan\left(x + \frac{\pi}{6}\right)$: $B = 1$, phase shift $= -\frac{\pi}{6}$ (left)

$$P = \pi$$

Parent asymptotes at $\pm\frac{\pi}{2}$, shifted left by $\frac{\pi}{6}$:

$$x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6} = \frac{\pi}{3} \qquad \text{and} \qquad x = -\frac{\pi}{2} - \frac{\pi}{6} = \frac{-3\pi - \pi}{6} = -\frac{2\pi}{3}$$

$$\boxed{P = \pi, \quad \text{Phase shift } \frac{\pi}{6} \text{ left,} \quad \text{Asymptotes at } x = \frac{\pi}{3} \text{ and } x = -\frac{2\pi}{3}}$$

(c) $y = -\tan(x) + 2$: $A = -1$, $B = 1$, $D = 2$

$$P = \pi, \qquad \text{Reflected, vertical shift} = 2$$

Asymptotes unchanged: $x = \pm\frac{\pi}{2}$.

$$\boxed{P = \pi, \quad \text{Reflected, Up 2,} \quad \text{Asymptotes at } x = \pm\frac{\pi}{2}}$$

(d) $y = 3\tan\left(2x - \frac{\pi}{2}\right)$: $A = 3$, $B = 2$, $C = \frac{\pi}{2}$

$$P = \frac{\pi}{2}, \qquad \text{Phase shift} = \frac{C}{B} = \frac{\pi/2}{2} = \frac{\pi}{4} \text{ (right)}$$

Asymptotes: solve $2x - \frac{\pi}{2} = \pm\frac{\pi}{2}$:

$$2x = \pi \;\Rightarrow\; x = \frac{\pi}{2} \qquad \text{and} \qquad 2x = 0 \;\Rightarrow\; x = 0$$

$$\boxed{P = \frac{\pi}{2}, \quad \text{Phase shift } \frac{\pi}{4} \text{ right,} \quad \text{Asymptotes at } x = 0 \text{ and } x = \frac{\pi}{2}}$$

2. Three key points.

(a) $y = \tan(x)$: standard key points.

$$\boxed{\left(-\frac{\pi}{4},\, -1\right), \quad (0,\, 0), \quad \left(\frac{\pi}{4},\, 1\right)}$$

(b) $y = 2\tan(x)$: multiply y-values by 2.

$$\boxed{\left(-\frac{\pi}{4},\, -2\right), \quad (0,\, 0), \quad \left(\frac{\pi}{4},\, 2\right)}$$

(c) $y = \tan(2x)$: period $= \dfrac{\pi}{2}$, key x-values are half the standard ones.

$$\boxed{\left(-\frac{\pi}{8},\, -1\right), \quad (0,\, 0), \quad \left(\frac{\pi}{8},\, 1\right)}$$

3. $y = -2\tan\left(x - \dfrac{\pi}{4}\right) + 1$

(a)

$$P = \pi, \qquad \text{Phase shift} = \frac{\pi}{4}\ \text{(right)}, \qquad \text{Vertical shift} = 1, \qquad \text{Reflected}$$

$$\boxed{P = \pi, \quad \text{Phase shift } \frac{\pi}{4} \text{ right}, \quad \text{Up } 1, \quad \text{Reflected}}$$

(b) Parent asymptotes at $\pm\dfrac{\pi}{2}$, shifted right by $\dfrac{\pi}{4}$:

$$x = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} \qquad \text{and} \qquad x = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$$

$$\boxed{x = -\frac{\pi}{4} \text{ and } x = \frac{3\pi}{4}}$$

(c) Shift parent key points right by $\dfrac{\pi}{4}$, then apply $y \to -2y + 1$:

Parent $(x,\, y)$	Shifted right $\frac{\pi}{4}$	$y \to -2y + 1$
$\left(-\frac{\pi}{4},\, -1\right)$	$(0,\, -1)$	$-2(-1) + 1 = 3$
$(0,\, 0)$	$\left(\frac{\pi}{4},\, 0\right)$	$-2(0) + 1 = 1$
$\left(\frac{\pi}{4},\, 1\right)$	$\left(\frac{\pi}{2},\, 1\right)$	$-2(1) + 1 = -1$

$$\boxed{(0,\, 3), \quad \left(\frac{\pi}{4},\, 1\right), \quad \left(\frac{\pi}{2},\, -1\right)}$$

4. $P = \dfrac{\pi}{|B|} = \dfrac{\pi}{3}$

$$|B| = \frac{\pi}{\pi/3} = 3$$

$$\boxed{|B| = 3}$$

5. The tangent function has no amplitude because it has no maximum or minimum value. Unlike sine and cosine, which are always bounded between -1 and 1, tangent increases without bound as it approaches each vertical asymptote. Because there is no finite distance from the midline to any peak, amplitude is not defined for tangent.

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